

1979

A long-range planning model for utility expansion using goal programming

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A LONG-RANGE PLANNING MODEL FOR UTILITY
EXPANSION USING GOAL PROGRAMMING.

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A long-range planning model for utility
expansion using goal programming

by

David William Poock

A Dissertation Submitted to the
Graduate Faculty in Partial Fulfillment of the
Requirements for the Degree of
DOCTOR OF PHILOSOPHY

Department: Industrial Engineering
Major: Engineering Valuation

Approved:

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CHAPTER I. INTRODUCTION

There is an increasing interest in the development of utility corporate models throughout the world for studying financial plans and system expansion plans. When one considers the rapidly changing spectrum of problems faced by utility system planners and managements, the need for development of these planning tools is almost self-evident. System expansion plans can no longer be viewed in isolation, but must be studied with the realization that these plans will effect the entire company and its financial needs and future structure. In recent years the financial constraints and realities of corporate existence have caused engineers to consider these aspects of their plans in addition to using their conventional economic evaluation procedures. The usual, or classical, measures of corporate economic and financial performance are revenue requirements and net income, respectively. Stable corporate performance with changing conditions and plans plus considerations involving the utility's liquidity (i.e., cash and investment) may be extremely important constraints in planning evaluation. Model studies of planning situation permit the incorporation of all of these important considerations.

Many different model schemes and programs have been formulated. The major difference between various models appears

to be the degree of emphasis placed on modeling cash management. Utility finance officers seem to favor monthly financial models with more detail on short-term cash flows while engineering planners are more interested in the long-range aspects of their plans. Both types of models have their place and both have been effectively used for planning evaluations. The difficulty with the monthly model is in the detail of input required and the time and effort necessary to establish a model. Long-range models do not provide the necessary information needed for cash flow.

This research describes a new approach to long-range utility corporate models designed specifically to facilitate use in planning situations. The model is written to minimize the detail of input data required and to facilitate comparison between alternative plans.

It must be remembered that the use of mathematical models is only a part of an overall set of approaches that have an effect upon managerial actions. So, despite the great amount of research accomplished in the area of mathematical models, it should be emphasized that models are not very useful except when used in conjunction with a broader, comprehensive approach to decision analysis.

Multiple Objectives in the Decision Process

Traditional economic theory presumes that the decision-maker is rational. Thus, when the decision-maker is placed in a profit making setting, economic factors alone supposedly motivate him. Today, researchers see the decision-maker as one who must perceive the alternatives available, assign some system of payoffs to these alternatives, and be able to decide which of these sets of payoffs is best for the firm. This process is often complicated by the existence of multiple, conflicting objectives. In determining the payoffs available from the various alternatives under consideration, it should be realized that complete attainment of objectives is usually not possible. Consequently, selection of alternatives becomes much more difficult. Therefore, the existence of multiple objectives affects the decision-making process in any organization.

Although the supposed objective of a profit-making enterprise is often expressed as that of maximizing either profit or shareholders' wealth, in practice the existence of other objectives may be as important, if not more so. Poque and Lall (1974) conducted a study suggesting that the objectives of a firm are many and that profit, the traditional economic objective, is not the most important. In their study, for example, social responsibility and the desire to

satisfy the customer preceded the profit motive. The authors also concluded from their research that tools involving single criteria are not adequate and that multi-objective models need to be developed. Similarly, in their work on behavioral theory as it applies to the decision process, Cyert and March (1963) provide a clear picture of the importance of dealing with multiple conflicting objectives. Their theory of the firm regards decision-making not so much as an optimizing process, but rather as one in which a set of constraints is satisfied to produce goal attainment. Cyert and March identify five major goals of the firm—production, inventory, sales, market share, and profit. The decision process, then, undertakes to satisfy these goals. This approach contrasts somewhat with the traditional economic theory of profit maximization, and it presents a more realistic picture of the problems faced by organizations. These studies indicate the necessity of recognizing the existence of multiple objectives in the decision-making process.

Uniqueness of a Public Utility

That an energy crisis exists is easy to claim. Just what the problem is, can be more difficult to argue. The United States is consuming about 90 QUADS (90×10^{15} Btu) per year and domestically producing about 65 QUADS (Bailey,

1978). This represents a shortfall of almost 1/3 of our needs. By 1990, the United States will demand 145 QUADS and produce only 90. Thus, the demand is growing at a rate of about 4% per year while supply is growing at only 2.5% per year.

There are a wide variety of technological fixes to the supply problem. The United States can burn more coal, find more gas, create safe ways to use nuclear power, and harness the sun. All of these options require huge outlays in capital dollars. In Table 1.1 the forecasted demand for capital needed by the electric industry is shown. At present, that industry consumes about 20 billion dollars per year. This represents 12 percent of the nonresidential investment capacity of the U.S. By the year 2000, Table 1.1 suggests the electric generating industry will require 60 billion dollars which is 20 percent of extrapolated U.S. capacity. In Iowa this represents construction of approximately 25 new medium-sized power plants with a cost of 12 billion dollars.

For public utilities, regulation has led to a modification of traditional approaches to capital budgeting. In the "traditional view" of the capital budgeting process, the firm takes on projects so long as their rate of return exceeds the cost of capital. According to traditional regulatory theory, this conceptual model is not generally

Table 1.1. Forecasted annual capital expenditures for the electric power industry in millions of dollars (Bailey, 1978)

Year	Dollars ^a
1976	21,196
1977	20,802
1978	22,031
1979	21,889
1980	24,383
1981	25,687
1982	27,088
1983	29,154
1984	31,386
1985	31,848
1986	35,486
1990	43,427
1995	52,889

^aAll figures in 1976 dollars.

applicable to utility companies. In the regulatory process, a target, or allowed rate of return, is specified. This return is either implicitly or explicitly, recognized as being a point (perhaps midpoint) within a range of rates of return frequently called the "zone of reasonableness". If "good" capital investments cause the actual rate of return

to exceed the upper end of this range, then a rate reduction is ordered to drive rates back down to the target.

Figure 1.1 shows the rate of return pattern facing a typical utility company when: 1) inflation is driving costs up constantly, 2) prices, which are set by regulatory action, are increased at discrete intervals, and 3) regulatory lag is present. At point A the actual ROR (rate of return) penetrates the lower control limit, prompting the company to ask for a rate hearing, which occurs at point B. At point C an order is issued permitting the company to raise rates and the rate increase takes effect at point D. The actual ROR does not return to the target level. The cost figures generally used in the point B rate cases are those of the most recent past year. If inflation continues, by the time the new rates take effect, the cost figures are outdated. Hence, the calculated utility rates are too low to return the ROR on investment to the target level.

Brigham and Pettway (1973) conducted a survey of capital budgeting by utilities. The results indicated that 40% of the companies surveyed have been subject to capital rationing. Of the firms, 89% indicated that in response to funds shortage they would apply for a rate increase. The utilities were questioned about their dividend policies. According to the respondents, only about one-third of the

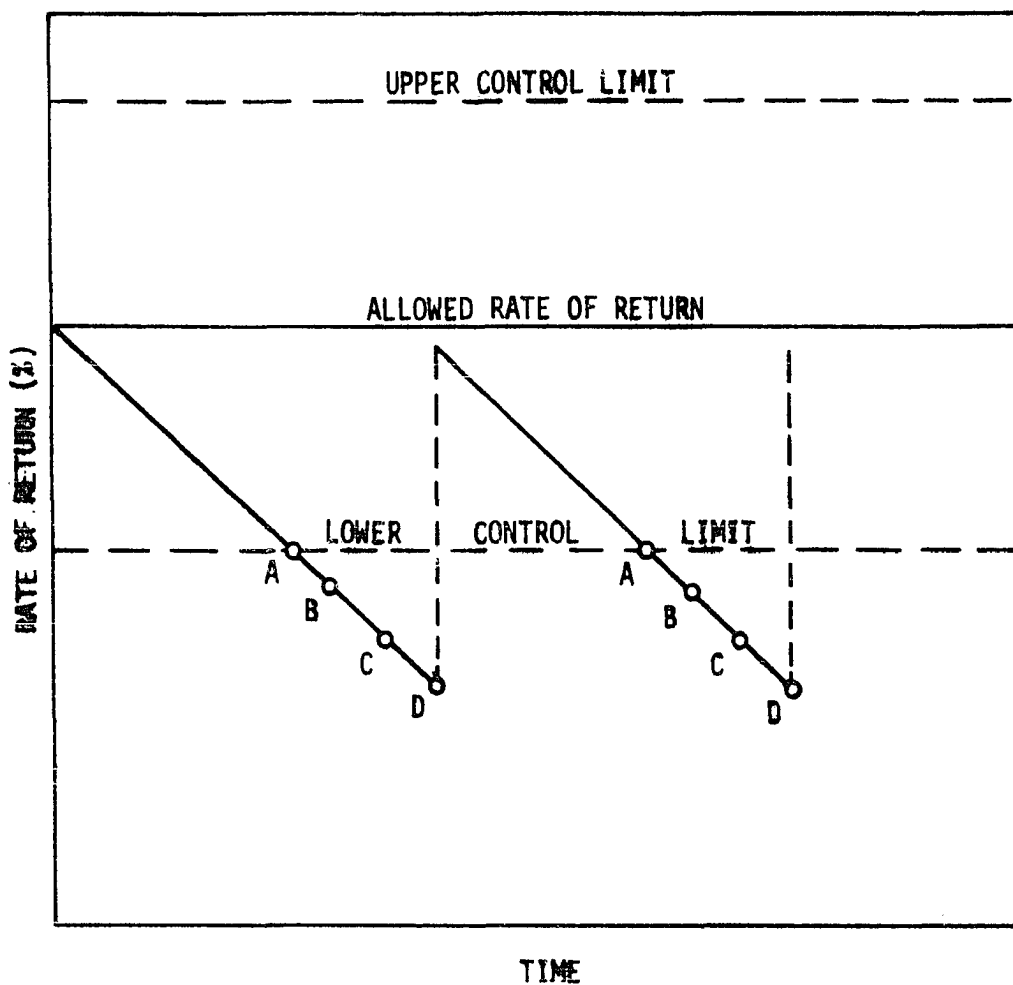


Figure 1.1. Rate of return under inflationary conditions with regulatory lag

utility companies' dividend policies are adjusted to changing investment opportunities or capital market conditions.

Under inflation the established pattern of rate regulation has not worked out as utility theory assumes, and, as a result, the utility companies have been placed in a difficult position. On the one hand, they must make whatever investment is necessary to meet service demands. At the same time, the companies must generate the cash necessary to maintain the current dividend policy.

Research Objectives

The purpose of this research is to develop a goal programming model for electric utilities and to demonstrate its application potential to managerial decision-making. In presenting the model, the approach adapts methods already developed for electrical expansion models.

The dissertation consists of six chapters. Chapter I has discussed the importance of developing a goal programming model for an electric utility. A brief review of the literature for both goal programming and expansion models for electric utilities is contained in Chapter II. Chapter III presents the goal programming model that has been adapted to utility expansion planning. A solution procedure, including a computer program, for goal programming is presented in Chapter IV. Chapter V contains the results of applying the

model under various assumptions. As is traditional, Chapter VI discusses conclusions reached and makes suggestions for further research.

CHAPTER II. LITERATURE REVIEW

Literature pertaining to this research can be conveniently divided into two categories: 1) Multiple Objective Programming, and 2) Studies Related to Investment Planning and Utility Expansion.

Multiple Objective Programming

Multiple objective programming deals with optimization problems with two or more objective functions. The general form with n decision variables, m constraints and k objectives is

$$\begin{aligned} \text{Minimize } & [Z_1(x_1, x_2, \dots, x_n), \\ & [Z_2(x_1, x_2, \dots, x_n), \dots, \\ & [Z_k(x_1, x_2, \dots, x_n)] \end{aligned} \quad (2.1)$$

subject to

$$f_i(x_1, x_2, \dots, x_n) \geq 0 \quad i = 1, 2, \dots, m \quad (2.2)$$

$$x_j \geq 0 \quad j = 1, 2, \dots, n$$

where $Z_1(\cdot)$, $Z_2(\cdot)$, ..., $Z_k(\cdot)$ are the k individual objective functions. Note that the individual objective functions are merely listed in (2.1); they are not added, multiplied, or combined in any way. The method of solution can best be

described by the information flows in the process. Information flows are important because they determine the role that the analyst must play in the planning process.

For purposes of this research, it is sufficient to conceive of two types of information flows: 1) from decision-maker to analyst ("top-down") and 2) from analyst to decision-maker ("bottom-up"). The decision-maker-analyst flow occurs when decision-makers explicitly articulate preferences so that a best-compromise solution may be identified. This is referred to as goal programming. The analyst-decision-maker flow contains results about noninferior alternatives, their impact on the objectives, and the tradeoffs among the objectives. This is called generating techniques.

Iterative Techniques

Generating techniques emphasize the development of information about a multiple objective problem that is presented to a decision-maker in a manner that allows the range of choice and the tradeoffs among objectives to be well-understood. The information flow is of the bottom-up variety. Analysts apply a generating technique to find an exact representation or an approximation of the noninferior set (Cohen, 1978).

Optimality plays an important role in the solution of single-objective problems. It allows the analyst and

decision-makers to restrict their attention to a single solution or a very small subset of solutions from among the much larger set of feasible solutions. A new concept called noninferior will serve a similar but less limiting purpose for multiple objective problems (Klahr, 1958).

The idea of noninferiority is very similar to the concept of dominance. Noninferiority is called "nondominance" by some mathematical programmers (Hannan, 1978), "efficiency" by statisticians and economists (Dyer, 1972), and "pareto optimality" by welfare economists (Cohen, 1978). Suppose three solutions in a two-objective problem are given as in Table 2.1. Alternative C is dominated by A and B because both of these alternatives yield higher values of both objectives, Z_1 and Z_2 . A solution that is dominated in this manner is termed inferior. Solutions that are not dominated are noninferior. Thus, for example, alternatives A and B in Table 2-1 are noninferior. To get a bit more formal, noninferiority can be defined in the following way:

Table 2.1. An example of noninferiority

Alternative	Z_1	Z_2	
A	20	15	Noninferior
B	22	11	Noninferior
C	12	8	Inferior

A feasible solution to a multiple objective programming problem is noninferior if there exists no other feasible solution that will yield an improvement in one objective without causing a degradation in at least one other objective (Luenberger, 1969).

In recent years the problem of generating a subset of the noninferior solutions has been approached from the viewpoint of vector function minimization (Philip, 1972). Evans and Steuer (1973) used the revised simplex method for generating the noninferior set. These methods are based on parametric considerations.

Zeleny (1974) developed a multicriteria simplex method for generating all noninferior solutions from a given set of nondominated extreme points.

Preference-oriented Techniques

Techniques that incorporate preferences share the analytical goal of the generating methods: analysis of a multiple objective problem without explicit consideration of the political dynamics of the problem. Unlike the implicit treatment of preferences by the generating methods, however, preference-oriented techniques require that decision-makers articulate their preferences and pass that information on to the analyst. The two basic methods for articulation of preferences are noniterative and iterative approaches. Goal programming is an example of the former and the step method is an example of the latter.

The concept of goal programming was first introduced by Charnes and Cooper (1961) as a means of treating linear-programming problems with multiple conflicting objectives. In their approach, the researchers recognized that complete goal attainment is not always possible. Since such a condition indicates that no convex set exists, the authors suggested a scheme to incorporate deviations from goals into a linear programming objective function with the goal of minimizing these deviations.

Unfortunately, the notation used by those involved in goal programming is, by no means, standardized. The general goal programming mathematical model is expressed in the following notation (Ignizio, 1978):

Find $\bar{x} = x_1, \dots, x_j, \dots, x_J$ so as to minimize:

$$\bar{a} = \{g_1(\bar{n}, \bar{p}), \dots, g_k(\bar{n}, \bar{p}), \dots, g_K(\bar{n}, \bar{p})\} \quad (2.3)$$

such that:

$$f_i(\bar{x}) + n_i - p_i = b_i \text{ for all } i = 1, \dots, m \quad (2.4)$$

and

$$\bar{x}, \bar{n}, \bar{p} \geq 0 \quad (2.5)$$

where:

x_j is the j th decision variable,

\bar{a} is denoted as the achievement function; a row vector measure of the attainment of the objectives or constraints at each priority level,

$g_k(\bar{n}, \bar{p})$ is a function (normally linear) of the deviation variables associated with the objectives or constraints at priority level k ,

k is the total number of priority levels in the model,

b_i is the right-hand side constant for goal (or constraint) i ,

$f_i(\bar{x})$ is the left-hand side of the linear or nonlinear goal or constraint i ,

n_i is the negative deviation from goal i , and

p_i is the positive deviation from goal i .

Under such a formulation, given any type of goal or constraint, it is desired to minimize the nonachievement of that goal or constraint by minimizing specific deviation variables. Table 2.2 summarizes the approach taken to accomplish this desire.

Table 2.2. Model formulation

Goal or constraint type	Processed goal or constraint	Deviation variables to be minimized
$f_i(\bar{x}) \leq b_i$	$f_i(\bar{x}) + n_i - p_i = b_i$	p_i
$f_i(\bar{x}) \geq b_i$	$f_i(\bar{x}) + n_i - p_i = b_i$	n_i
$f_i(\bar{x}) = b_i$	$f_i(\bar{x}) + n_i - p_i = b_i$	$n_i + p_i$

The deviation variables at each priority level, k , are included in the function $g_k(\bar{n}, \bar{p})$ and ordered in the achievement vector \bar{a} , according to their respective priority.

Developments in goal programming were made by Ijiri

(1965). Although Ijiri's work was more directly concerned with the field of accounting control, it resulted in several contributions to the study of goal programming. One of these was the idea of preemptive priority factors in a linear programming format. In this model, the deviations from goals, as ranked by the priority factors, are minimized in the solution process. Secondly, Ijiri proposed the generalized inverse method as a solution technique. In this technique, the square root of the sum of squares of goal deviations are to be minimized.

Although Ijiri proposed the general inverse procedure as a solution method for goal programming, it was not until Lee (1972) developed the modified simplex technique that goal programming found an efficient solution method. In this method, the basic simplex procedure of linear programming is utilized to minimize the deviational variables of the goal. Deviational variables are ranked according to preemptive priority factors so that during the solution process the goals are considered in order of their priorities. In addition, a weighting method is allowed to incorporate cardinal values to goals at a given priority level. A computer program for the modified simplex method of goal programming has been widely used.

Much of the recent work in goal programming has been in

the area of applications. The earliest application was a study of advertising media planning by Charnes, Cooper, De Voe, Learner and Reinecke (1968). In this model, goals were established for percentages of audience segments reached by different types of advertising.

Lee and Nicely (1974) presented a case study demonstrating how goal programming may be used in market planning. The subject of the case was color television sets. The model analyzed the effects of promotion on rates of return, the number of television sets leased and personnel policies.

Several goal programming studies have been made in the area of financial decision-making. In a capital budgeting application, Lee and Lerro (1974) pointed out the advantages of incorporating multiple objectives in the selection of capital investments. Taylor and Keown (1978) formulated a goal programming model for project selection where both profit and nonprofit motivated projects are in competition for scarce resources.

A comprehensive list of areas where goal programming has been applied can be found in Kornbluth (1973) and Ignizio (1978).

Procedures that incorporate preferences operate with local approximations of a decision-maker's preferences. The locally approximated preference information is articulated by the decision-maker in response to local information about

the noninferior set generated by the analysis. Benayoun, de Montgolfier and Tergny (1971) developed the step method. The linear multiple objective program is optimized with respect to each goal individually. The decision-maker and the analyst determine the appropriate goals to relax until a satisfactory solution is obtained. The decision rule is to minimize the maximum deviation from the best possible goal.

Investment Planning

Project planning is concerned with choices among alternative investment opportunities. These investment opportunities include not only business decisions, such as which plant to build and hence, which new technology to adopt, but also the amounts to be spent by government on roads, education, research, military facilities and the like.

One of the earlier works dealing with capital budgeting was a linear programming model by Weingartner (1963). His model employed an objective function composed of net present values of investment proposals from which will be selected, under constrained financing, that combination bringing the highest return to the firm.

Baumol and Quandt (1965) developed a seemingly different programming model which attempted to maximize shareholder wealth by providing the investor with an optimal

dividend stream. This implies an objective function where future dividend payments are discounted using marginal utility as the appropriate discount factor and available cash as the constraint. Despite the introduction of utility, Meyers (1974) demonstrated that there is little difference in meaning between this model and Weingartner's model.

Since the problem of capital budgeting is one that affects the entire structure of the modern corporation, Spies (1974) formulated a model which incorporates the dynamic nature of the problem. The capital budget was broken down into five basic components: dividends, short-term investment, gross long-term investment, debt financing and new equity financing.

The previously described models avoided a more realistic model of the capital budgeting problem. The reason is that such traditional formulations are restricted to the consideration of only a single objective function whereas, in most real-world problems there are usually several, conflicting objectives that are desirable to the decision-maker. A representative sample of the goal programming models for capital budgeting are: Hawkins and Adams (1974); Ignizio (1976a); Keown and Martin (1977 and 1978); Lee and Lerro (1974); Sartoris and Spruill (1974); and Taylor and Keown (1978).

Bussey (1978) demonstrated all the models are only correct under the assumption of perfect capital markets which are summarized as follows:

1. financial markets are perfectly competitive;
2. there are no transaction costs;
3. information is complete, costless and available to all; and
4. all individuals and firms are able to borrow and lend on the same terms.

It is the fourth assumption which causes the failure of the net present value criterion. However, Bussey did demonstrate that a goal programming model would still be valid.

Bernhard (1971) and Cooley, Roenfeldt and Chew (1975) identified the discount rate as a second problem with Weingartner's and Baumol and Quandt's models. With capital rationing and inflation, the same discount rate can not be used for the planning horizon.

The models used in the optimal expansion of an electrical supply system can be classified as mathematical programming models covering a particular subsystem. Generating facilities are the most frequently considered subsystems. Bessiere (1970) formulated a nonlinear model while Juseret (1978) solved the optimization problem using convex programming.

Le (1977) formulated a large scale chance-constrained linear programming model to determine the optimal expansion

over a planning horizon. Petersen (1973) attacked the same problem using a dynamic programming methodology.

Shelton (1977) constructed a mixed integer programming model to determine the optimal expansion of a distribution subsystem.

Anderson (1972) provides an excellent state-of-art discussion of the various models used in the planning of the expansion of a power system. He illustrates several models which could be used. The models possess two characteristics: they only investigate a subsystem and they use a net present value as the criterion for the objective function.

CHAPTER III. A GOAL PROGRAMMING MODEL FOR
ELECTRIC UTILITY PLANNING

Planning may be defined as formulating, evaluating and choosing between the various courses of action being considered. In an electrical supply system this process consists primarily of determining the sequence of expansion with regard to generating units, transmission lines, transformers, circuit breakers and other major plant components. The course of action must be determined in such a way that the system is in a position to meet future electrical demands with an adequate security of supply combined with the lowest possible capital and operating costs and with existing financial options duly taken into account.

The planning of the electrical supply system raises special problems. Plants must continually be installed to meet the increasing demand for electricity but capital requirements for expansion are very large. The "leadtime" required between making the decision and the commissioning of a plant is relatively long. The potential capacity available from the supply system must exceed the simultaneous sum of the consumers' demands at all times if restrictions are to be avoided. Abnormalities developing in one part of the system are immediately felt to a greater or lesser extent throughout the system. In the planning of an electrical supply system

expansion critical issues are encountered such as increasing capital costs, financial and environmental restraints, and increasing fuel costs. If these issues are compounded by the effects of changing technologies and the limited availability of resources, it becomes clear that a comprehensive analysis of the future outlook for an electrical supply system is an enormously complex undertaking.

In general, the aim of power system planning is to provide a pattern of expansion which will ensure that sufficient plant is available to supply the forecasted load with an adequate level of reliability, and that this pattern of expansion is the lowest cost alternative of those available.

Brigham and Pettway (1973) demonstrated that a utility is confronted with the conflicting goals of consumers' demand and stockholders' dividend. This tradeoff between timing of investments and replacements versus maintaining dividends at a constant rate is represented in a goal programming framework.

Goal Programming

That organizations have a number of objectives is commonly accepted. Moreover, problems arise because these objectives often conflict. Thus, achievement of some objectives may be possible only by not attaining others. In

mathematical terms, no convex set of feasible solutions exists. Goal programming offers one method of resolving these conflicting objectives. The technique has been accepted by academicians and practitioners as a major quantitative tool to be used in the treatment of multiple objectives.

The general model for a goal programming problem follows:

$$\text{Minimize } \bar{a} = \{g_1(\bar{n}, \bar{p}), g_2(\bar{n}, \bar{p}), \dots, g_k(\bar{n}, \bar{p})\} \quad (3.1)$$

subject to

$$\sum_{j=1}^n a_{ij}x_j + n_i - p_i = b_i \quad i = 1, 2, \dots, m \quad (3.2)$$

$$x_j, n_i, p_i \geq 0$$

where

n_i is the negative deviation variable,

p_i is the positive deviation variable,

$g_k(\bar{n}, \bar{p})$ is a linear function of the deviation variables,

\bar{a} is an ordered vector whose components are the $g_k(\bar{n}, \bar{p})$ functions,

a_{ij} is the coefficient of x_j in goal i , and

b_i is the right-hand-side value of goal i .

In the solution process of goal programming, it is important to understand that the goals are not necessarily being optimized, but, rather, are being satisfied. Of course, goal programming can be so formulated to achieve an

optimization. This is different in concept from linear programming where the single objective function is optimized to get the "best" solution. In real world situations, however, the typical decision problem may in fact be to operate within a rather narrow set of (possibly incompatible) constraints, and linear programming cannot handle this type of problem on a satisfactory basis for several reasons. First, linear programming does not easily allow an exact ordinal ranking of objectives. This may be achieved only by arriving at a system of weights for the various goals. However, arriving at this set of weights is difficult, and of course the approach contradicts the spirit of an ordinal ranking of priorities (Lee, 1972). Also, unless these goals are incorporated into the objective function rather than the constraint set, infeasibility may result, which renders a solution impossible. Zeleny (1974) offers a contrasting approach for the solution of goal programming problems. Here, a linear programming solution is used where the objective function is composed of a set of objectives with a constraint set similar to linear programming.

Another major disadvantage of linear programming is the unidimensionality of the objective function (Lee, 1972). That is, the objective function must be expressed in terms of the same units, whether dollars or hours, since strict

comparability does not exist among quantities expressed in dissimilar units. On the other hand, in goal programming, the objective function tries to satisfy the constraint set, which may be composed of any quantifiable measurements. Thus, it does not present any difficulty if some goals are expressed in terms of dollars or hours while others are in units of output.

In decision analysis applying goal programming, the decision-maker must decide upon his ordering of priorities and be able to express them in quantitative terms. For example, he may decide that stabilization of employment is preferable to meeting a certain level of profit. If this is the case, he would try to attain a specific employment level at a higher priority than the profit goal. Using priority levels that differ forces the solution process to consider the goals on an ordinal basis, so that the employment level is achieved as nearly as possible before the profit is considered. Yet, goal programming is flexible enough to accommodate a cardinal ranking if it is desired.

Perhaps the most difficult part of the process occurs in determining the priority structure of the goals. This is the responsibility of the decision-maker. However, an important part of the goal programming process is to evaluate goal underachievement after a solution is reached. Thus, the soundness of the decision-maker's priority structure can

be explicitly evaluated. Three types of solutions can be attempted in the goal programming model: 1) the amount of resources required to attain the desired goals; 2) the level of goal attainments using the given resources; and 3) the level of goal attainment under varying goal requirements and resource capabilities (Ignizio, 1976). Using this information, it is relatively easy to analyze the effects of changes upon the system.

Mathematical Model for Electric Utility Planning

In the past decade numerous attempts have been made to apply mathematical optimization and simulation methods in the development of models for planning the expansion of the electricity supply system (Anderson, 1972; Bessiere, 1970; Jusseret, 1978; and Petersen, 1973). Presently, there is no electrical supply undertaking of any size which does not use mathematical models for carrying out generation planning, transmission planning and financial planning. Most of the models are characteristically developed and used for solving specific planning tasks for a subsystem. Hence, the solution obtained may correspond to sub-optimal solutions. The modeling system described here forms a framework for the discussion of the principles of planning an integrated electrical supply system is given in Figure 3.1. This figure

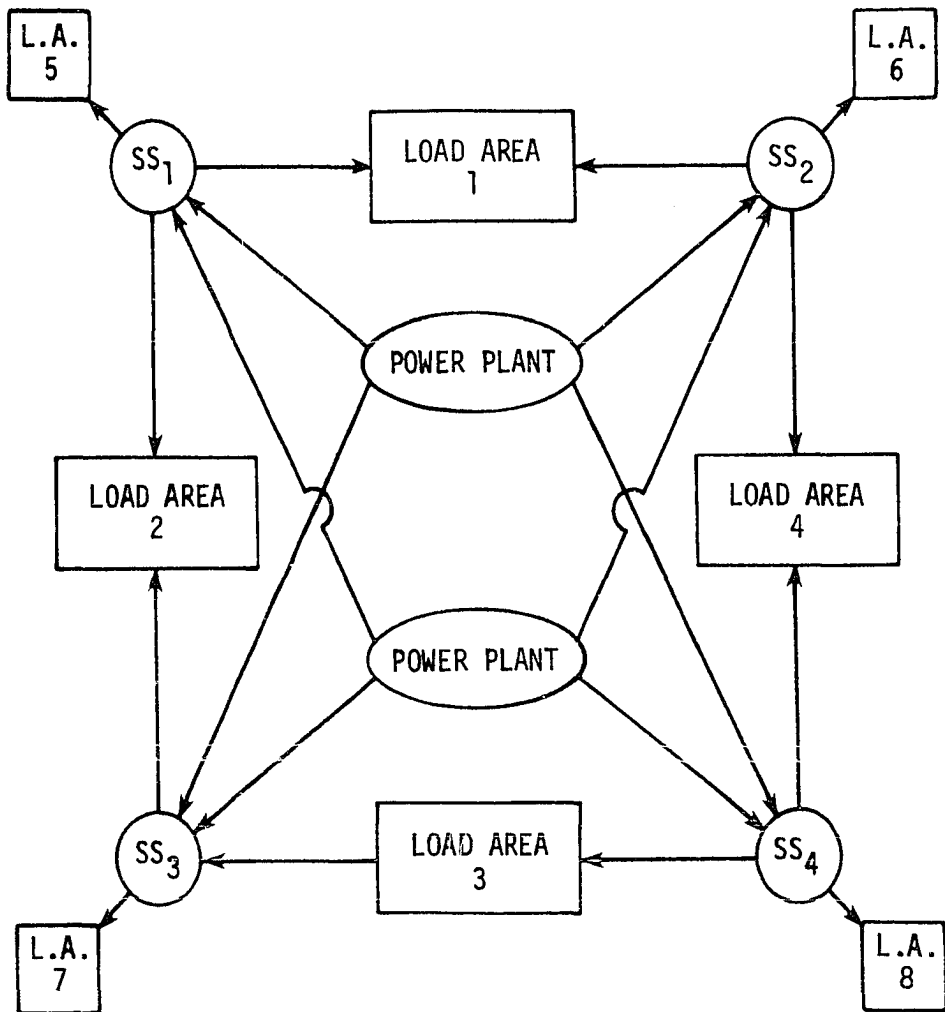


Figure 3.1. Electrical supply system

represents a typical electrical supply system. As illustrated, the system is composed of two generating plants, of four substations, and of six customer demand areas.

The load duration curve is a device used in electric utility industries to show the number of hours for a period of time, say a year, that various loads are served.

The calculation of optimal operating schedules and costs is complicated by the variance of power demand, which varies throughout the day and year (Figure 3.2). The operating costs are the area under this curve weighted at each time interval, w_t , by the fuel costs and the output of the plant during that interval. Usually the calculation of operating costs is simplified by constructing a curve known as the load duration curve. This curve is constructed from the demand curve (Figure 3.2) by rearranging each load for each time interval w_t to occur in descending order of magnitude (Figure 3.3).

The load duration curve makes integration of cost easier because it can be represented by a simpler function than the curve in Figure 3.2.

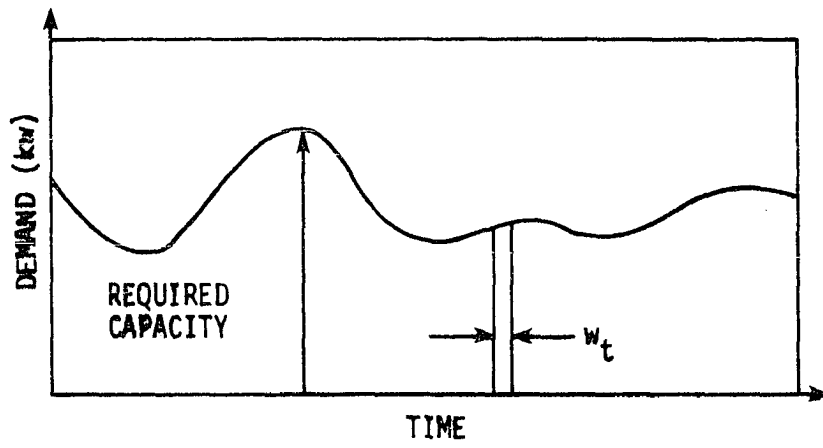


Figure 3.2. Power demand

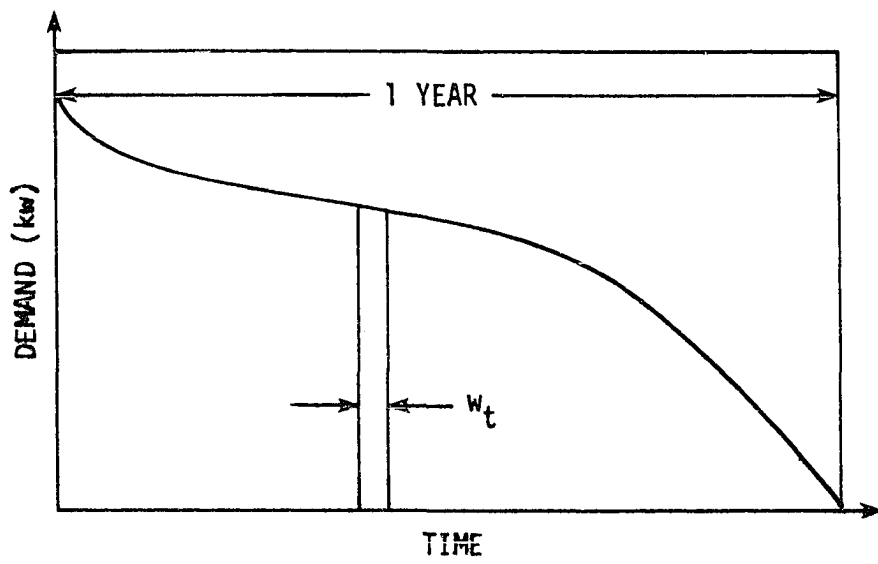


Figure 3.3. Load duration curve

Description of the Goal
Programming Model

Before discussing the decision variables and the constraints in the model, several assumptions about the system are listed as follows:

1. The quantities demanded are assumed to be exogeneous. This is the most practical way to treat interactions of demand and supply when formulating an investment program.
2. The formulation is deterministic. Allowances are made for uncertainties in demand and plant availability, but in the simple form of margins of spare capacity.
3. There is no discussion of terminal conditions.
4. Finally, the electricity supply system is assumed to be operating in a stable condition. That is, there are no transient or maintenance conditions that would cause downtime.

The subscripts (lower case) and decision variables (upper case) used in this model are as follows:

- l - load area ($l = 1, 2, \dots, L$),
- f - type of fuel ($f = 1, 2, \dots, F$),
- k - type of plant ($k = 1, 2, \dots, K$),
- y - years in study ($y = 1, 2, \dots, Y$),

- s - each period y is divided into $s = 1, 2, \dots, S$ subperiods (seasons),
 d - demands in each period s ($d = 1, 2, \dots, D$),
 n - number of substations ($n = 1, 2, \dots, N$),
 m - type of transformers ($m = 1, 2, \dots, M$),
 c - number of feeder circuits at substation n ($c = 1, 2, \dots, C$),
 p - type of pollution discharge ($p = 1, 2, \dots, P$),
 v - vintage of a power plant or a transformer ($v = 0, 1, \dots, Y$),
 PS - installed plant size of a power plant, (kw)
 OC - operating capacity of a power plant in year y (kw)
 GO - generated output of a power plant (kw),
 TC - transmission capacity of a power plant (kw),
 FC - fuel consumed at a power plant (Btu),
 NT - new transformers installed at a substation (MVA),
 RT - removed transformers at a substation (MVA),
 CD - cash dividends paid to stockholders (constant dollars),
 CB - cash borrowed by the firm (constant dollars), and
 CL - cash lent by the firm (constant dollars).

As an example, these subscripts and decision variables would be combined in the following form:

$FC(f, k, v, d, s, y)$ - the quantity of fuel f consumed at plant k , vintage v , month d , season s , and year y .

The input data that the decision-maker must provide is as follows:

- LAD - load area demand (kw),
- MT - maximum size of transformers at each substation (MVA),
- FCC - feeder circuit capacity at each substation (kw),
- e - energy conversion of fuel into electrical energy (kw/Btu),
- df - pollution discharge factor from fuel (particle/Btu),
- FL - limitations on fuel available (Btu),
- EP - environmental pollution limit (particles),
- w_t - width of time interval of block d on the load duration curve,
- CCP - cash cost per unit of initial capacity of a power plant (constant dollars/kw),
- CCT - cash cost per unit of transformer capacity at a substation (constant dollars/kw),
- PC - production costs (excluding fuel costs) per unit of energy output (constant dollars/kw),
- CT - cash cost per unit of transmission capacity (constant dollars/kw),
- CF - cash cost per unit of fuel consumed, units (constant dollars/Btu),
- COE - cash operating expenditures (constant dollars),
- COF - cash operating fuel expenditures (constant dollars),
- COP - cash operating pollution expenditures (constant dollars),
- MC - minimum fixed cash balance (constant dollars),

- CA - cash available (constant dollars),
 BL - borrowing limit (constant dollars),
 l - lending rate (decimal), and
 b - borrowing rate (decimal).

The constraints are divided into three sections. The first 8 restrictions applied to the power plants; while the next 5 constraints are applicable to the substations. The final set of 5 constraints are the financial constraints.

1. Operating capacity of a power plant in year y must be less than installed plant size.

$$\begin{aligned} OC_{k,v,y} &\leq PS_{k,v} && \text{for } y = 1, \dots, Y && (3.3) \\ & && v = 1, \dots, Y && \\ & && k = 1, \dots, K && \end{aligned}$$

2. Operating capacity in any year must be less than the operating capacity in the previous year.

$$\begin{aligned} OC_{k,v,y+1} &\leq OC_{k,v,y} && \text{for } y = 1, \dots, Y && (3.4) \\ & && v = 1, \dots, Y && \\ & && k = 1, \dots, K && \end{aligned}$$

3. Generated output must be less than its operating capacity

$$\sum_{l=1}^L GO_{k,v,d,s,y,l} \leq a_{k,v,d,s,y} OC_{k,v,d,s,y} \quad (3.5)$$

$$\text{for } k = 1, \dots, K$$

$$v = 1, \dots, Y$$

$$d = 1, \dots, D$$

$$s = 1, \dots, S$$

$$y = 1, \dots, Y$$

$$0 < a_{k,v,d,s,y} < 1$$

4. Operating capacity must be greater than the peak load required at a substation by a margin g_y

$$\sum_{v=1}^y \sum_{k=1}^k OC_{k,v,y} \geq (1+g_y) \sum_{n=1}^N \sum_{k=1}^K \sum_{v=1}^y GO_{k,v,n,d,s,y} \quad (3.6)$$

for $d = 1$ (peak)

$$s = 1, \dots, S$$

$$y = 1, \dots, Y$$

$$0 < g_y < 1$$

5. Transmission capacity between power plants and substations must be sufficient to carry peak load by a margin h_y

$$\sum_{v=1}^y \sum_{k=1}^k TC_{k,v,n} \geq (1+h_y) \sum_{v=1}^y \sum_{k=1}^K GO_{k,v,n,d,s,y} \quad (3.7)$$

for $n = 1, \dots, N$

$d = 1$ (peak)

$$s = 1, \dots, S$$

$$y = 1, \dots, Y$$

$$0 < h_y < 1$$

6. Conversion of fuel into electrical energy must be greater than the generated output

$$\sum_{f=1}^F e_{f,k,v}^{FC} \geq \sum_{n=1}^N GO_{k,v,n,d,s,y} w_d \quad (3.8)$$

$$\text{for } k = 1, \dots, K$$

$$v = 1, \dots, Y$$

$$d = 1, \dots, D$$

$$s = 1, \dots, S$$

$$y = 1, \dots, Y$$

7. Amount of fuel f consumed must be less than than the available supply for each w_d , s , and y .

For w_d :

$$\sum_{v=1}^Y \sum_{k=1}^K FC_{f,k,v,d,s,y} \leq FL_{f,d,s,y} \quad (3.9)$$

$$\text{for } f = 1, \dots, F$$

$$d = 1, \dots, D$$

$$s = 1, \dots, S$$

$$y = 1, \dots, Y$$

For period s :

$$\sum_{v=1}^Y \sum_{k=1}^K \sum_{d=1}^D FC_{f,k,v,d,s,y} \leq \sum_{d=1}^D FL_{f,d,s,y} \quad (3.10)$$

$$\text{for } f = 1, \dots, F$$

$$s = 1, \dots, S$$

$$y = 1, \dots, Y$$

For period y:

$$\sum_{s=1}^S \sum_{d=1}^D \sum_{v=1}^Y \sum_{k=1}^K FC_{f,k,v,d,s,y} \leq \sum_{s=1}^S \sum_{d=1}^D FL_{f,d,s,y} \quad (3.11)$$

8. Pollution particles (p) must be less than an upper limit that may be harmful to the environment during each w_d, s, y .

For w_d :

$$\sum_{v=1}^Y \sum_{k=1}^K \sum_{f=1}^F FC_{f,k,v,d,s,y}^{df} \leq EP_{p,d,s,y} \quad (3.12)$$

for $p = 1, \dots, P$

$d = 1, \dots, D$

$s = 1, \dots, S$

$y = 1, \dots, Y$

for S:

$$\sum_{v=1}^Y \sum_{k=1}^K \sum_{f=1}^F \sum_{d=1}^D FC_{f,k,v,d,s,y}^{df} \leq \sum_{d=1}^D EP_{p,d,s,y} \quad (3.13)$$

for $p = 1, \dots, P$

$s = 1, \dots, S$

$y = 1, \dots, Y$

For y :

$$\sum_{v=1}^Y \sum_{k=1}^K \sum_{f=1}^F \sum_{d=1}^D \sum_{s=1}^S FC_{f,k,v,d,s,y}^{df} f_{p,k,v,d,s,y} \leq \sum_{s=1}^S \sum_{d=1}^D EP_{p,d,s,y} \quad (3.14)$$

for $p = 1, \dots, P$

$y = 1, \dots, Y$

9. Transformer capacity must be greater than the circuit loads at each substation.

$$\sum_{v=1}^Y \sum_{n=1}^N \sum_{m=1}^M [NT_{m,n,v,d,s,y}^{-RT} + ET_{m,n,v,d,s,y}] \geq LAD_{l,d,s,y} \quad (3.15)$$

for $l = 1, \dots, L$

$d = 1, \dots, D$

$s = 1, \dots, S$

$y = 1, \dots, Y$

10. The number of transformers at a substation must be less than the allowed maximum number.

$$\sum_{m=1}^M (NT_{m,n,v,d,s,y}^{-RT} + ET_{m,n,v,d,s,y}) \leq MT_{n,v,d,s,y} \quad (3.16)$$

for $n = 1, \dots, N$
 $v = 1, \dots, Y$
 $d = 1, \dots, D$
 $s = 1, \dots, S$
 $y = 1, \dots, Y$

11. A nonexistent transformer must not be removed

$$\sum_{a=1}^Y [NT_{m,n,v,d,s,y}^{-RT_{m,n,v,d,s,y}} + ET_{m,n,v,d,s,y}] \geq 0 \quad (3.17)$$

for $m = 1, \dots, M$
 $n = 1, \dots, n$
 $v = 1, \dots, y$
 $d = 1, \dots, D$
 $s = 1, \dots, S$
 $y = 1, \dots, Y$

12. A transformer must not be removed from a substation unless it is being moved to another substation.

$$\sum_{b=1}^n [NT_{m,n,v,d,s,y}^{-RT_{m,n,v,d,s,y}}] \geq 0 \quad (3.18)$$

for $m = 1, \dots, M$
 $n = 1, \dots, N$
 $v = 1, \dots, y$
 $d = 1, \dots, D$
 $s = 1, \dots, S$
 $y = 1, \dots, Y$

13. The circuit loads in each load area must be greater than the total load in the area.

$$\sum_{n=1}^N \sum_{i=1}^c SC_{c,n,d,s,y} \geq LAD_{l,d,s,y} \quad (3.19)$$

$$\text{for } l = 1, \dots, L$$

$$d = 1, \dots, D$$

$$s = 1, \dots, S$$

$$y = 1, \dots, Y$$

14. At time y : the net cash outflow to projects (new power stations and new transformers); minus the cash inflow from time $y-1$ loans; plus cash outflow from time y loans; plus the cash outflow for repayment of time $y-1$ borrowing; minus the cash inflow from time y borrowing; plus the cash outflow for time y dividends payment must be as a sum less than or equal to the cash available.

$$\begin{aligned} & - \sum_{k,v,s,d,n} \{ CC_{k,v,s,d,y}^{PS} \} \\ & + CNT_{n,m,v,s,d,y}^{NT} \\ & - l_{y-1} (CL_{y-1} + c_y CB_{y-1} + MC_{y-1}) \\ & + (CL_{y-1} + c_y CB_y + M_y) \\ & + b_{y-1} CB_{y-1} - CB_y + CD_y \leq CA_y \end{aligned} \quad (3.20)$$

$$\text{for } y = 1, \dots, Y$$

$$0 < c_y < 1$$

15. The cash operating budget must be less than the total budget in any year.

$$\sum_{k,v,d,s} PC_{k,v,d,s,y}^{GO} \leq COE_y \quad (3.21)$$

for $y = 1, \dots, Y$

16. The cash expenditure for fuel must be less than the total budget for any year.

$$\sum_{f,k,v,s,d} CF_{f,k,v,s,d,y}^{FC} \leq COF \quad (3.22)$$

for $y = 1, \dots, Y$

17. The cash expenditure for environmental protection must be less than the total budget for any year.

$$\sum_{f,k,v,s,d} CE_{f,k,v,d,s,y}^{FC} \leq COP_y \quad (3.23)$$

18. The cash borrowed in any year must be less than the borrowing capacity.

$$CB_y \leq BL_y \quad (3.24)$$

for $y = 1, \dots, Y$

The model developed in Equations 3.3 through 3.24 can be used for any planning period that the decision-maker selects. For the system shown in Figure 3.1 and using a 20 year planning horizon, the total number of constraints would be 14,204 and the model would contain 15,370 decision variables.

In mathematical programming there is usually one objective function which the decision-maker either minimizes or maximizes. However, in goal programming, there is a series of objectives which the decision-maker ranks on an ordinal basis. For an electric utility, some of the goals might be as follows:

1. Maintain a given debt ratio,
2. Maintain growth in earnings,
3. Maximize cash inflows,
4. Spend a minimum amount on environmental protection,
5. Minimize capital budget overruns,
6. Minimize the fuel adjustment factor,
7. Minimize cash operating expenses,
8. Satisfy customer demands,
9. Maintain a minimum level of plant operation,
10. Minimize amounts of energy purchased, and
11. Minimize excess liquidity.

The decision-maker would then establish an aspiration level for each goal selected and an ordinal ranking of these goals. One possible ranking could be as follows:

- Priority 1: Goals 8 and 9,
- Priority 2: Goals 1, 2, 6 and 7,
- Priority 3: Goals 3, 4 and 5, and
- Priority 4: Goals 10 and 11.

Once the goals are ranked, the decision-maker would then form the achievement vector (Table 2.2). It should be recalled that the goals within a ranking must be commensurable but not across a ranking.

Model Characteristics

The above goal programming model represents an electricity supply system. The model includes the generating facilities, the transmission network, and the financial requirements. The decision-maker has tremendous latitude in defining the scope of the major components.

The model is ideally suited to investigate the tradeoffs that occur from various rankings of the goals. This would be extremely beneficial in cases involving governmental agencies. This will be explored in a limited fashion in Chapter V.

CHAPTER IV. SOLVING GOAL PROGRAMMING
MODELS

Goal programming is a methodology that allows the decision-maker to explicitly state and examine the various alternatives that are available. The solution of these models is illustrated in this chapter, via an example. A new computer program, which uses the ideas of revised simplex and compact storage in computers, is developed.

Formulation Example

Ace Electronics Incorporated manufactures two types of stereo headsets. One headset, the Deluxe, requires 1 hour in assembly, while the other, the Supreme, requires 2 hours assembly time. The normal assembly operation is limited to 40 hours per week. Marketing surveys indicate that no more than 30 Deluxe and 15 Supreme headsets should be produced each week. The net profit from the Deluxe model is \$8 each and is \$12 each from the Supreme model.

The company president has stated the following objectives in order of priority:

1. Maximize total profits,
2. Minimize overtime operation of the assembly line,
3. Sell as many stereo headsets as possible (this is not necessarily the same as maximizing profit).

The decision model is as follows:

Find x_1 and x_2 so as to minimize:

$$\bar{a} = \{(p_3+p_4), (p_2), (n_1), (n_3+1.5 n_4)\} \quad (4.1)$$

such that:

$$\begin{aligned} 8x_1 + 12x_2 + n_1 - p_1 &= 1000 & C_1 \\ x_1 + 2x_2 + n_2 - p_2 &= 40 & C_2 \\ x_1 + n_3 - p_3 &= 30 & C_3 \\ x_2 + n_4 - p_4 &= 15 & C_4 \end{aligned} \quad (4.2)$$

where:

x_1 = number of Deluxe headsets,

x_2 = number of Supreme headsets,

n_i = the amount of underachievement of goal i , and

p_i = the amount of overachievement of goal i .

That is, the first priority is to satisfy the absolute objective of never exceeding demand through minimization of p_3 and p_4 . Any solution in which both p_3 and p_4 are not zero is considered unimplementable. The second priority is given to minimization of overtime and is achieved by minimizing p_2 . The third priority is assigned to maximizing profits (minimize n_1). The fourth and final priority is to sell as many sets as possible by minimizing n_3 and n_4 . Since Supremes receive 1.5 times the profit of Deluxe models, more emphasis is placed on the minimization of n_4 .

Graphical Solution

The four constraints are plotted as straight lines in Figure 4.1. Note that only the decision variables (i.e., X_1 and X_2) are used in the plot. However, the effect of an increase in any deviation variable ($N_1, N_2, N_3, N_4, P_1, P_2, P_3,$ and P_4) is reflected by the arrows at each constraint line. The particular deviation variables to be minimized (i.e., those in the achievement vector) have been circled.

The graphical solution is demonstrated in Figures 4.1 through 4.5. An attempt is made to satisfy priority one goals. The solution space satisfying priority one is indicated by the cross-hatched area of Figure 4.2. Here both P_3 and P_4 are set to zero.

Next, an attempt is made to satisfy the priority two goal without degrading the solution to priority one. This can be accomplished by setting P_2 to zero. The solution to priority levels one and two is given in Figure 4.3.

If priority three is to be achieved, N_1 must be minimized. However, N_1 cannot be set to zero as this would degrade the solution at both priority one and two. The solution minimizing N_1 while not degrading P_1 and P_2 is given by point A in Figure 4.4. The value of priority level three at point A is 700, while the value at point B is 740.

Finally to achieve (as close as possible) priority 4,

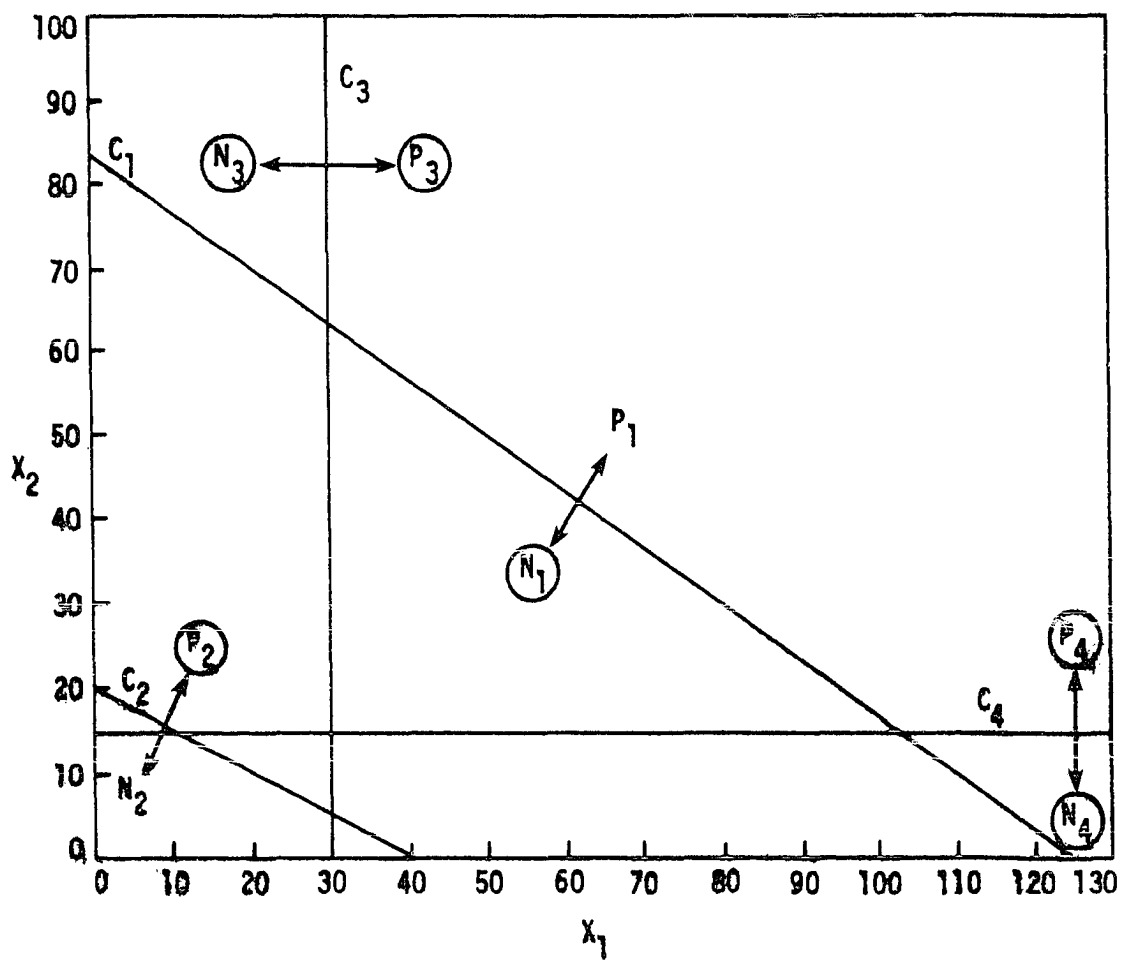


Figure 4.1. Graphical representation of formulation example

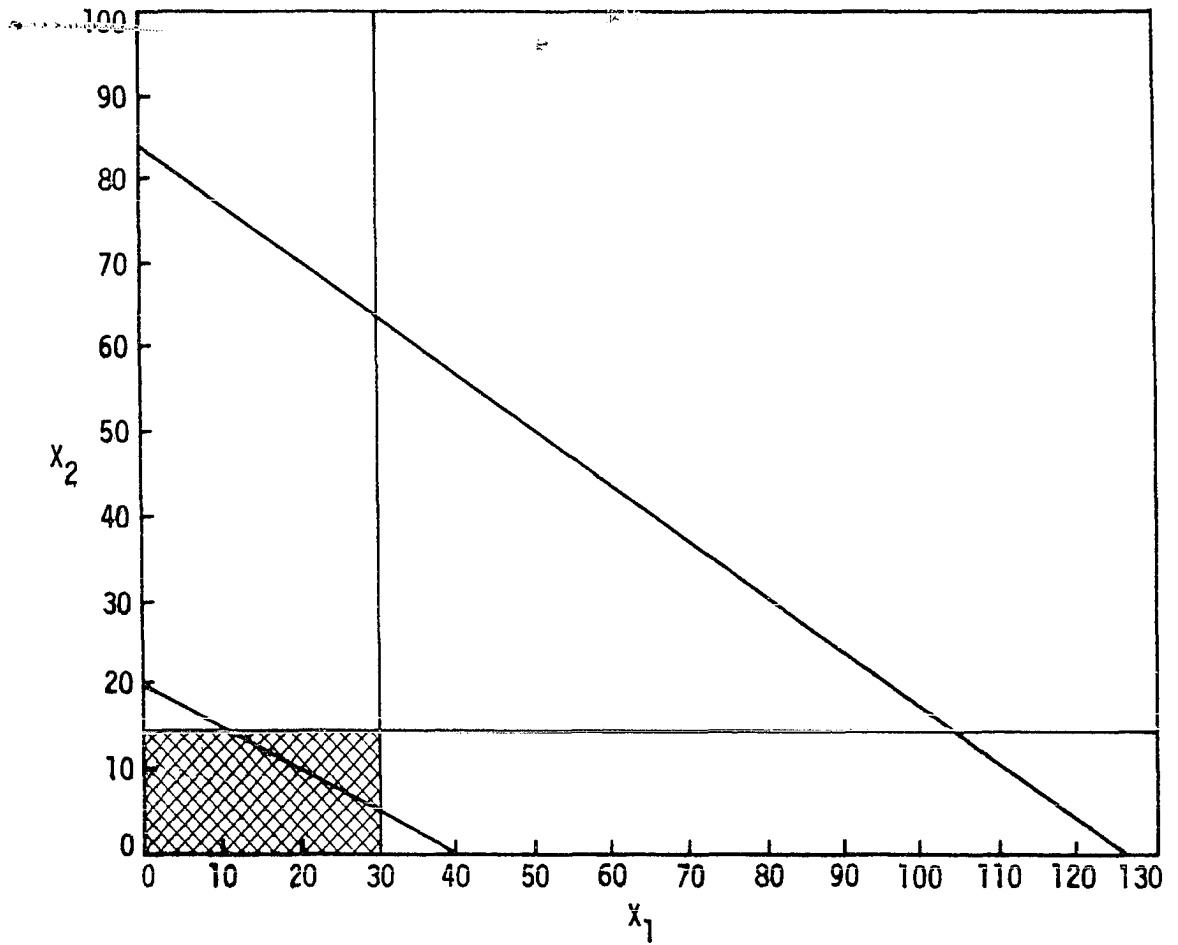


Figure 4.2. Solutions to priority level one

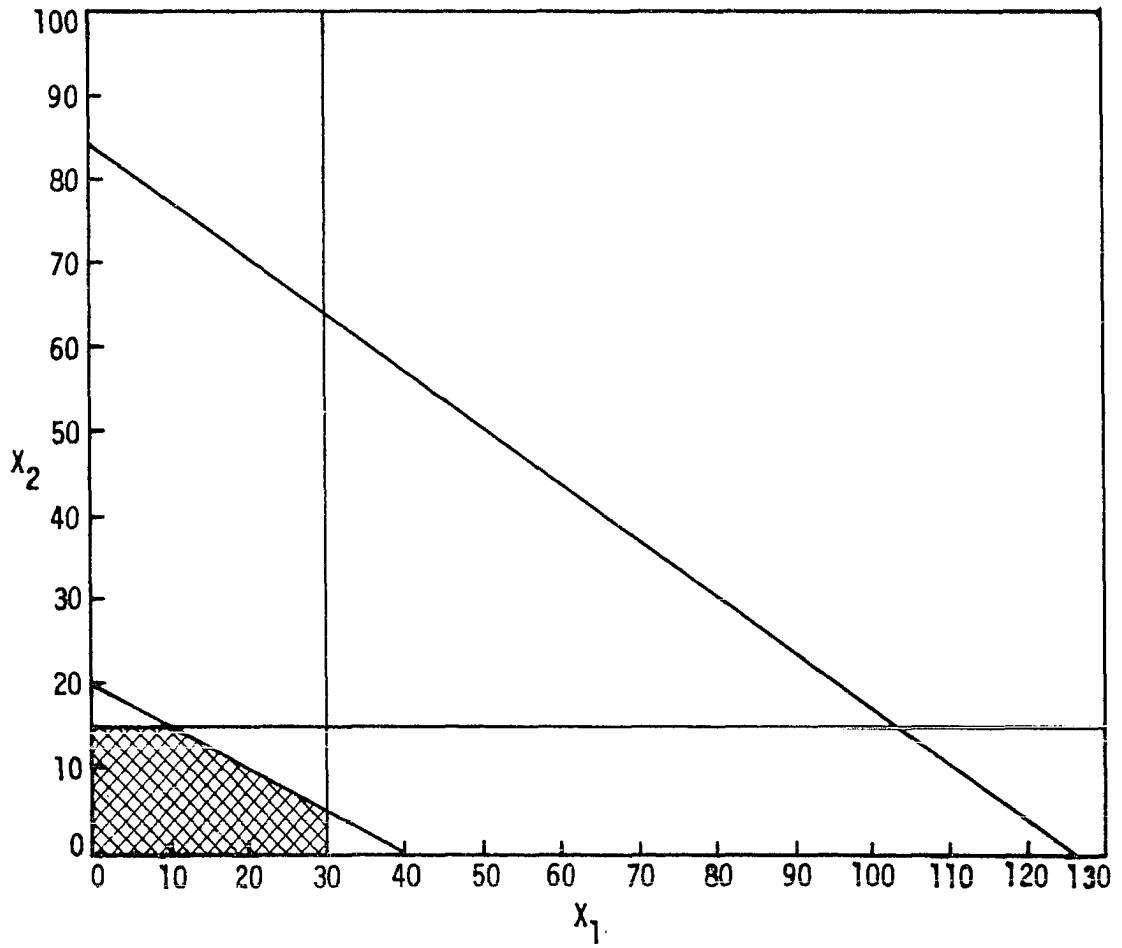


Figure 4.3. Solutions to priority levels one and two

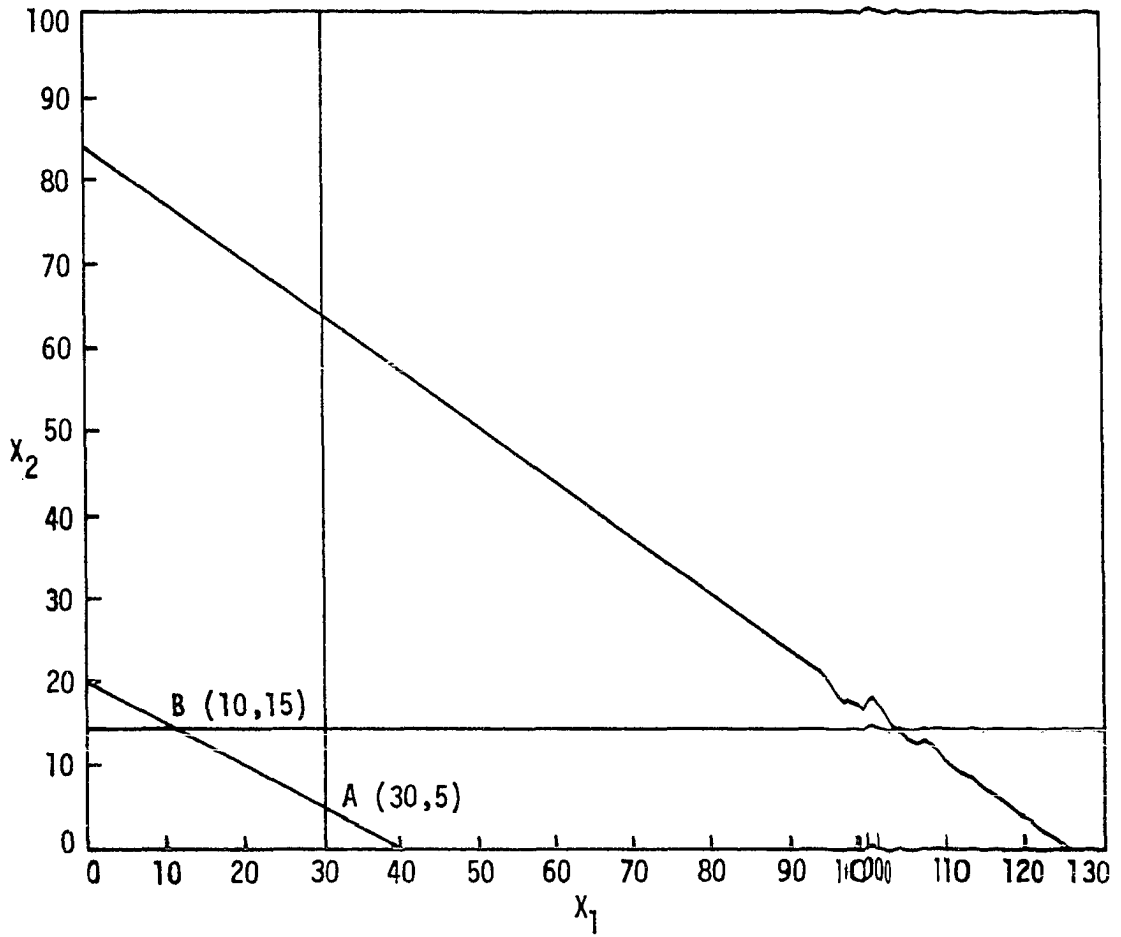


Figure 4.4. Solutions to priority levels one, two, and three

N_3 and N_4 must be minimized, but notice N_4 is considered 1.5 times as important. Consequently the final solution is the point shown in Figure 4.5. If N_4 was equal to zero, the solution for priority level three would be degraded.

Therefore, Ace Electronics Incorporated should manufacture 30 Deluxe headsets and 5 Supreme headsets per week. The firm would use no overtime and would receive \$300 in profits.

Revised Goal Programming

The graphical procedure is limited to small problems. A revised goal programming (RGP) procedure, which is based on the revised simplex procedure (Evans and Steuer, 1973), has been developed. Before considering the RGP procedure, define the following matrices:

TW is a $(k \times 2m)$ matrix composed of the weights given to the negative and positive deviates in the k priority levels,

a is a $(k \times 1)$ column vector composed of the values for the k priority levels,

X is a $((n+2m) \times 1)$ column vector; the first n components are the decision variables; the next m components are the negative deviates; the last m components are the positive deviates,

where:

$k = 1, 2, \dots, K$ (priority levels)

$m = 1, 2, \dots, M$ (number of constraints)

$n = 1, 2, \dots, N$ (number of decision variables)

Then the goal programming model (Equations 3.1 and 3.2) can

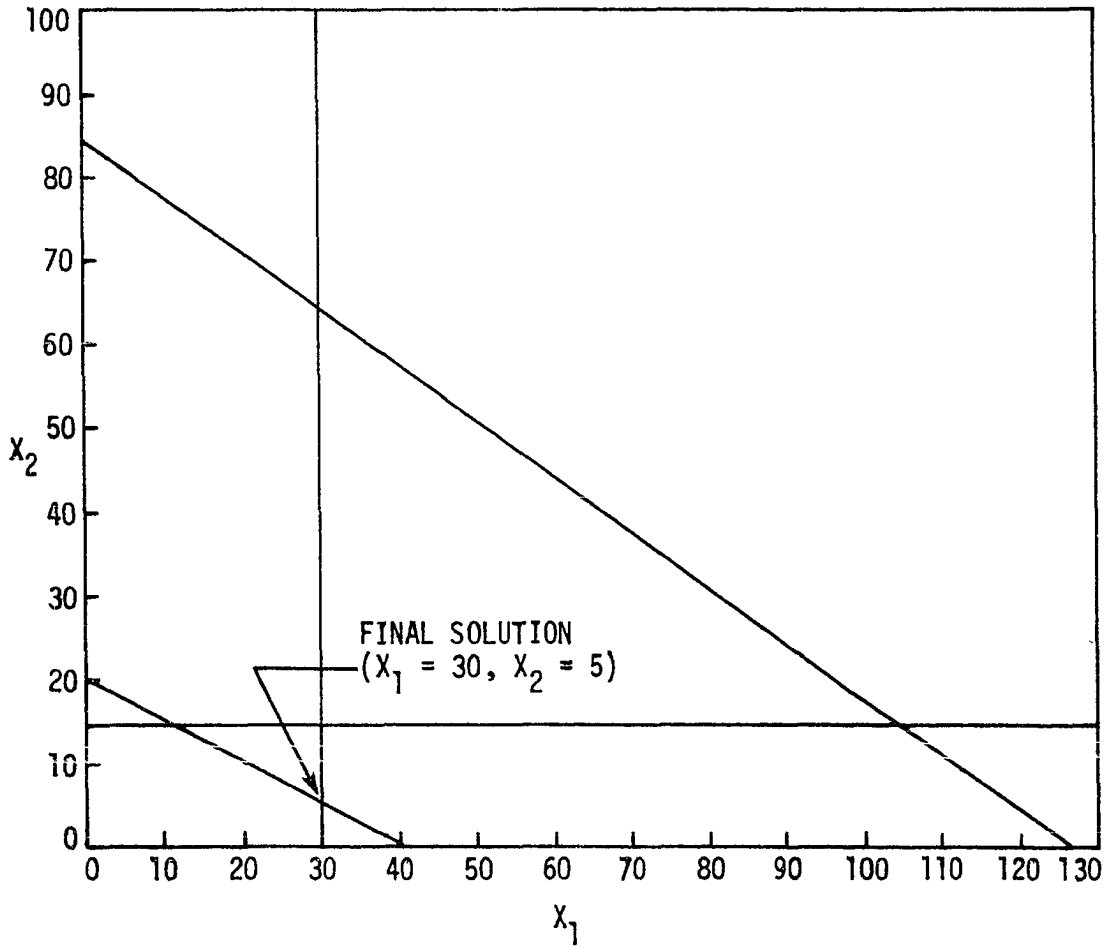


Figure 4.5. Solution to all priority levels

be written in the following form:

$$\begin{bmatrix} \bar{I}_{k \times k} & \bar{O}_{k \times n} & \bar{TW}_{(k \times 2m)} & \bar{a}_{k \times 1} \\ \bar{O}_{m \times k} & \bar{A}_{m \times n} & \bar{I}_{m \times m} - \bar{I}_{m \times m} & \bar{X}_{(n+2m) \times 1} \end{bmatrix} = \begin{bmatrix} \bar{O}_{k \times 1} \\ \bar{b}_{m \times 1} \end{bmatrix} \quad (4.3)$$

which has the following solution

$$\begin{bmatrix} \bar{a}_{k \times 1} \\ \bar{X}_{(n+2m) \times 1} \end{bmatrix} = \begin{bmatrix} \bar{I}_{k \times k} & \bar{TW}_B \bar{B}^{-1}_{k \times m} \\ \bar{O}_{m \times m} & \bar{B}^{-1}_{m \times m} \end{bmatrix} \begin{bmatrix} \bar{O}_{k \times 1} \\ \bar{b}_{m \times 1} \end{bmatrix} \\ = \begin{bmatrix} \bar{TW}_B \bar{B}^{-1} \bar{b} \\ \bar{B}^{-1} \bar{b} \end{bmatrix} \quad (4.4)$$

It should be noted that the elements in \bar{TW}_B are the weights of the basic variables in the TW matrix. It is only necessary to modify the usual simplex criterion of selecting the entering variable. The minimum ratio test remains in effect. The new rule is as follows:

Select the first a_k to minimize (attempt to force to zero). Select the nonbasic variable with the most positive coefficient to enter the basis. There must not be a negative coefficient, at a higher priority level, for the entering variable. Ties are broken arbitrarily. If all $a_k \leq 0$ or if no positive coefficient exists, stop.

At the beginning of cycle k , assume that \bar{B}^{-1} , the associated basic solution $\bar{X}_B = \bar{B}^{-1} \bar{b}$, and the data of the original problem (A , \bar{TW} , \bar{b}) are available. Cycle k proceeds as follows:

1. Compute the achievement vector

$$a = TW_B B^{-1} b \quad (4.5)$$

2. If all a_k equal zero, stop. The current basic solution is optimal.

3. If any $a_k > 0$, compute the coefficients of the nonbasic variables in the priority levels

$$TW_B B^{-1} b - TW_{NB} \quad (4.6)$$

4. For $a_k > 0$, select the nonbasic variable from (4.6) to enter the basis. Label that column s .

5. Compute

$$\frac{\bar{b}_r}{\bar{a}_{rs}} = \min_{\bar{a}_{is} > 0} \frac{\bar{b}_i}{\bar{a}_{is}} \quad (4.7)$$

where r denotes the leaving column.

6. Update the new inverse matrix and basic solution.
Return to step 1.

The above steps will now be applied to the problem that was formulated at the beginning of this chapter. The negative deviates will form the initial set of basic variables.

The first cycle is:

$$1. a = TW_B B^{-1} b = \begin{bmatrix} 0 \\ 0 \\ 1000 \\ 52.5 \end{bmatrix}$$

$$3. \quad TW_B B^{-1} A - TW_{NB} = \begin{bmatrix} 0 & 0 & 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 8 & 12 & -1 & 0 & 0 & 0 \\ 1 & 1.5 & 0 & 0 & -1 & -1.5 \end{bmatrix}$$

4. For a_3 , variable x_2 enters the basis.

5. Determine the minimum ratio: (1000/12, 40/2, 15/1)

6. Therefore, x_2 replaces n_4 and the new inverse matrix and solution are

$$B^{-1} = E_2 E_1 = \begin{bmatrix} 1 & 0 & 0 & -12 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$B^{-1} b = \begin{bmatrix} 820 \\ 10 \\ 30 \\ 15 \end{bmatrix}$$

The second cycle is:

$$1. \quad a = TW_B B^{-1} b = \begin{bmatrix} 0 \\ 0 \\ 820 \\ 30 \end{bmatrix}$$

$$3. \quad TW_B B^{-1} A - TW_{NB} = \begin{bmatrix} 0 & 0 & 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 8 & -12 & -1 & 0 & 0 & 12 \\ 0 & -1.5 & 0 & 0 & -1 & 0 \end{bmatrix}$$

4. For a_3 , variable x_1 enters the basis.
5. Determine the minimum ratio: $(820/8, 10/1, 30/1)$
6. Therefore x_1 replaces n_2 and the new inverse matrix and solution is

$$B^{-1} = E_3 E_2 E_1 = \begin{bmatrix} 1 & -8 & 0 & 4 \\ 0 & 1 & 0 & -2 \\ 0 & -1 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$B^{-1}b = \begin{bmatrix} 740 \\ 10 \\ 20 \\ 15 \end{bmatrix}$$

The third cycle is:

$$1. \quad a = TW_B B^{-1}b = \begin{bmatrix} 0 \\ 0 \\ 740 \\ 20 \end{bmatrix}$$

$$3. \quad TW_B B^{-1}A - TW_{NB} = \begin{bmatrix} 0 & 0 & -1 & 0 & -1 & -1 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ -8 & 4 & -1 & 8 & 0 & -4 \\ -1 & -0.5 & 0 & 1 & -1 & -2 \end{bmatrix}$$

4. For a_3 , variable n_4 enters the basis.

5. Determine the minimum ratio: $(740/4, 20/2, 15/1)$
6. Therefore, n_4 replaces n_3 and the new inverse matrix and solution are:

$$B^{-1} = E_4 E_3 E_2 E_1 = \begin{bmatrix} 1 & -6 & -2 & 0 \\ 0 & 0 & 1.0 & 0 \\ 0 & -0.5 & 0.5 & 1 \\ 0 & -0.5 & -0.5 & 0 \end{bmatrix}$$

$$B^{-1}b = \begin{bmatrix} 700 \\ 30 \\ 10 \\ 15 \end{bmatrix}$$

The fourth cycle is:

$$1. \quad a = TW_B B^{-1}b = \begin{bmatrix} 0 \\ 0 \\ 700 \\ 15 \end{bmatrix}$$

$$3. \quad TW_B B^{-1}A - TW_{NB} = \begin{bmatrix} 0 & 0 & 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ -6 & -2 & -1 & 6 & 2 & 0 \\ -.75 & -.25 & 0 & .75 & -.75 & -1.5 \end{bmatrix}$$

4. Since there are no possible entering variables for either a_3 or a_4 , stop. The solution given in step 6 of cycle 3 is the optimal solution.

Using the concept of linked lists, a computer program was developed to solve goal programming models.

Linked Lists

One of the main requirements for an efficient computer code is compact storage of the data. Linked lists are an efficient technique to accomplish this requirement. Linked lists enable the numerical values of the numbers to be stored in any order, the desired sequence of the numbers being determined by the linking technique. This linking procedure consists of allocating a storage location for the numerical value of each item and associating with this storage location the address for the numerical value of the next item. This technique was incorporated into the computer program and is illustrated by the following numerical example:

$$A = \begin{bmatrix} 1.5 & 0 & 0 & 3 \\ 0 & 1.3 & 0 & 4 \\ 1.2 & 0.5 & 2 & 0 \\ 1 & 0 & -7 & 0 \end{bmatrix}$$

The coefficient matrix A can be stored in a compact form using the linked lists techniques. In this case four arrays are necessary, these being VALUEA (numerical value of element in matrix A), IROWA (index row), ICAPA (index of column address pointer), and NOZEA (number of nonzero elements in each column of A). The four arrays are illustrated in Table 4.1. Any column of matrix A can be

Table 4.1. Storage of the A matrix

Location	1	2	3	4	5	6	7	8	9
VALUEA	1.5	1.2	1	1.3	0.5	2	-7	3	4
IROWA	1	3	4	2	3	3	4	1	1
ICAPA	1	4	6	8					
NOZEA	3	2	2	2					

reconstructed very simply. Consider the reconstruction of column 2. From the fourth array, the number of nonzero elements in column 2 is given by:

$$\text{NOZEA}(2) = 2$$

The location of the first element in column 2 is given by:

$$\text{ICAPA}(2) = 4$$

Therefore the nonzero elements of column 2 are given in locations 4 and 5 of the first array. This array indicates that the values of these elements and their corresponding row positions are:

$$\text{VALUEA}(4) = 1.3 \quad \text{IROWA}(4) = 2$$

$$\text{VALUEA}(5) = 0.5 \quad \text{IROWA}(5) = 3$$

In the computer program, AMAT, IAMAT1, and IAMAT2 are used for storage of the matrix of coefficients. The

elementary transformation columns are stored in the TMAT, ITMAT1, ITMAT2 matrices, while the weights in the achievement vector are stored in ZMAT, IZMAT1, and IZMAT2. A complete listing of the computer program, as well as input requirements, is given in Appendix B.

Test Cases

To test the efficiency of the computer program, several test cases were compared against a standard computer program (Lee, 1972). The results are summarized in Table 4.2.

Several comments can be made pertaining to the results obtained in the test cases. The first observation is that there is a noticeable decrease in the number of iterations required to solve a problem using the RGP program. A second comment relates to the CPU time. In all cases, the CPU time was less for the RGP program than for Lee's program. As the sparsity increases, the difference in CPU time increases. This is to be expected since the RGP program is written to handle sparse matrices. The last example in the table is a transportation problem which has a high degree of sparsity. A final comment about the two programs is the core size needed for the programs. The RGP program requires only 128K while Lee's program requires 256K.

Table 4.2. Comparison of Lee's program and the RGP program

Number of objectives	Number of variables	Number of constraints	Sparsity	Lee		RGP	
				Iterations	CPU	Iterations	CPU
2	2	2	0%	5	0.52	2	0.48
2	2	3	0%	4	0.55	1	0.49
2	3	5	23.4%	4	0.64	2	0.53
4	4	7	53.6%	10	0.88	4	0.78
3	12	10	66.7%	24	2.24	16	1.65

CHAPTER V. A CASE STUDY OF THE GOAL
PROGRAMMING MODEL

The model formulated in Chapter III will be solved using the RGP program developed in Chapter IV. Several test cases will be considered with a discussion of the results.

Input Data

For a 20-year planning horizon, the model developed in Chapter III (Equations 3.2 through 3.24) contains 14,204 constraints and 15,370 decision variables. It was decided to reduce the size of the model to a more manageable level. The simplifying assumptions were:

1. The planning horizon would be a 5-year period.
2. The two initial generating plants (300 mw and 500 mw) would serve only one substation which provides service to only one load area.
3. Three types of fuel (coal, oil, and gas) are available.
4. There is only one season in each year.
5. The lending and borrowing rates are constant throughout the planning horizon.
6. Environmental factors are eliminated.

With these assumptions, Equations 3.7, 3.12, 3.13, 3.14,

3.15, 3.16, 3.17, 3.18, and 3.23 are not needed in the model. As a result, the model now contains 78 constraints and 221 decision variables.

The historical data were obtained from Federal Power Commission (1975 and 1978), Edison Electric Institute (1976), and Le (1977). From these sources, an analysis was made on the data to estimate future operating parameters. An inflation rate of 8% was assumed to convert all future dollars to constant dollars. The results are given in Tables 5.1 and 5.2.

Priority Levels

In a goal programming model, there is no single objective function. Instead, the decision-maker must establish several goals which are then ranked on an ordinal basis. In addition, the decision-maker must set an aspiration level for each goal. For the model developed in this research, the four goals that were investigated and the aspiration level for each goal were:

1. Generated output of 35,480 mw,
2. Dividends paid of \$5,430,000,
3. Fuel consumed of 18,985,226 Btu's, and
4. Cash borrowed of \$50,800,000.

The generated output goal represents the requirement of the

Table 5.1. Input data for goal programming models, Part I

Year	Demand (10^9 kw)	Available fuel (10^{11} Btu's) ^a	Fuel cost (\$/kw)	Capital cost per unit of capacity (\$/kw)	Production cost (\$/kw)
1	3.645	c - 25.930	c - 0.013	137	4.95
		o - 0.017	o - 0.025		
		g - 8.046	g - 0.008		
2	4.878	c - 27.581	c - 0.018	165	6.15
		o - 0.029	o - 0.029		
		g - 8.094	g - 0.010		
3	6.528	c - 29.592	c - 0.022	225	7.34
		o - 0.024	o - 0.036		
		g - 8.207	g - 0.024		
4	8.735	c - 32.543	c - 0.022	320	7.90
		o - 0.024	o - 0.039		
		g - 8.352	g - 0.041		
5	11.691	c - 32.953	c - 0.025	375	8.54
		o - 0.026	o - 0.042		
		g - 8.444	g - 0.045		

^ac - coal, o - oil, g - gas.

Table 5.2. Input data for goal programming model, Part II^a

Year	Debt limit	Cash available	Operating expenditures	Fuel expenditures	Working capital	Minimum cash balance
1	205	36.8	1.267	4.091	12.9	20.5
2	205	37.9	1.315	4.231	13.2	20.5
3	210	39.1	1.420	4.431	13.8	21.0
4	215	38.2	1.430	4.651	13.5	21.5
5	215	39.5	1.541	4.848	14.3	21.5

^aAll numbers in millions of dollars.

utility to satisfy customers' demand while consuming no more than 18,985,226 Btu's. The utility would also like to pay dividends of \$5,430,000 and borrow no more than \$50,800,000 during the 5-year planning horizon.

To show the effect of various rankings, four cases were investigated. Table 5.3 lists the combinations that were considered in this research.

Table 5.3. Four test cases using four priority levels

Case	Priority			
	1	2	3	4
1	Energy Generated	Fuel Consumed	Cash Borrowed	Cash Dividends
2	Energy Generated	Cash Dividends	Fuel Consumed	Cash Borrowed
3	Energy Generated	Cash Borrowed	Fuel Consumed	Cash Dividends
4	Cash Dividends	Energy Generated	Fuel Consumed	Cash Borrowed

Results Using the RGP Program

The first case was solved using the computer program in Lee (1972) while the remaining cases were solved using the RGP program. Lee's program took 15.8 minutes of CPU time while the RGP program consumed 9.7 minutes, a reduction of 38.6%. This is to be expected since the matrix of

coefficients is 83% sparse.

The results are summarized in Table 5.4. A number means that the utility has failed to achieve that particular goal. For example, the priority level 4 in case 1 represents the cash dividends and has a value of \$410,213. This represents the amount by which the utility failed to pay dividends of \$5,430,000 during the 5-year planning horizon.

A second example is priority level 3 in case 3 which represents fuel consumed. The utility has an aspiration level of burning 18,985,226 Btu's during the 5-year planning horizon. The utility actually consumed 24,259,389 Btu's or a 27.8% increase in the planning value.

Table 5.4. Results of four cases using the RGP program

Case	Priority			
	1	2	3	4
1	0	0	35,100,500	410,213
2	0	1,011,512	0	45,100,000
3	0	1,253,798	5,274,163	310,492
4	0	0	2,176,934	25,162,746

Similar discussions can be made pertaining to each of the test cases given in Table 5.4. Tables 5.5 through 5.8 illustrate how various rankings can effect the timing of the decision variables.

In all cases, the total new plant construction was 1165 mw. However, the timing and the size of the plant was greatly affected by the ranking scheme. Case 2 had the largest variation in plant size with a low value of 400 mw in year 1 and a high value of 765 mw in year 4. This was also the only case in which construction was undertaken in year 1. None of the four cases had construction in year 5. Customers' demand was satisfied in all cases even with the various sizes and timing of the power plants.

The cash dividend policy was not completely achieved in three cases. Case 4, in which cash dividends had the highest priority, was the only case which satisfied the policy. In the other cases, the shortage range from \$310,492 (case 3) to \$1,011,512 (case 2). It is informative to investigate the variability of the cash dividends. The range on the individual cash dividends was a low of \$269,564 (case 1) to a high of \$406,920 (case 4). It should be noted that case 4 was the only case in which the total dividends paid match the utility's objective.

The utility's policy of borrowing only \$50,800,000 during the 5-year period was never achieved. Case 3, with the

Table 5.5. List of important decision variables, case 1

Year	Plant size (mw)	Cash dividends (dollars)	Cash borrowed (dollars)	Fuel consumed (10^6 Btu's)
1	0	1,076,100	10,245,617	3.399
2	650	813,814	13,783,931	3.570
3	0	1,050,750	25,243,948	3.782
4	515	946,649	25,380,847	4.091
5	0	1,132,474	11,246,157	4.142

Table 5.6. List of important decision variables, case 2

Year	Plant size (mw)	Cash dividends (dollars)	Cash borrowed (dollars)	Fuel consumed (10^6 Btu's)
1	400	937,142	27,691,345	3.399
2	0	1,057,143	10,615,032	3.222
3	0	851,742	22,549,633	3.649
4	765	761,937	24,217,246	4.091
5	0	810,524	10,826,744	4.623

Table 5.7. List of important decision variables, case 3

Year	Plant size (mw)	Cash dividends (dollars)	Cash borrowed (dollars)	Fuel consumed (10^6 Btu's)
1	0	1,026,514	10,245,133	3.121
2	0	1,101,850	13,679,105	4.752
3	575	854,490	12,187,938	5.201
4	590	998,245	7,469,111	4.879
5	0	1,138,409	8,472,511	6.305

Table 5.8. List of important decision variables, case 4

Year	Plant size (mw)	Cash dividends (dollars)	Cash borrowed (dollars)	Fuel consumed (10^6 Btu's)
1	0	953,106	11,204,636	3.694
2	675	1,001,493	25,198,504	4.205
3	490	882,958	16,213,100	3.984
4	0	1,310,459	13,005,487	5.206
5	0	1,281,984	10,341,019	4.073

borrowing limit at priority level 2, was closest to attainment of the goal. The difference was only \$1,253,798.

The largest deviation (\$45,100,000) occurred in case 2 in which the 765 mw power plant was constructed. Case 2 also had the largest range (\$17,076,313) while case 3 had the smallest range (\$6,209,994).

The underachievement of the fuel limitations occurred in cases 3 and 4. In case 3, an additional 5,274,163 Btu's were required while in case 4 an additional 2,176,934 Btu's were required. Even though cases 1 and 2 met the utility's policy, the range for case 1 was 746,673 Btu's and the range for case 2 was 1,400,317 Btu's. In both cases, the largest amount of fuel required in any one year occurred in year 5.

The results from these four cases clearly indicate the tradeoffs that a utility must make in long range planning. The principle conflict occurs between borrowing funds for new plant and maintaining cash dividends at a stable level.

CHAPTER VI. SUMMARY, CONCLUSIONS, AND
RECOMMENDATIONS

In this chapter a summary of the problem studied, the technique used, and the results obtained in this research is presented. Conclusions regarding the desirability of the technique and the usefulness of the results are then discussed. Finally, recommendations concerning extensions of the present investigation are considered.

Summary

The decision-maker concerned with long-range planning must consider the tradeoffs among the various options. Goal programming is a method for handling tradeoffs in a planning environment. This methodology allows the decision-maker to rank, on an ordinal basis, various objectives and examine the conflict among the various goals.

A goal programming model for an electric public utility was developed. For a 20-year planning horizon, the model contains 14,204 constraints and 15,370 decision variables. The size of the model was reduced to 78 constraints and 221 decision variables. The four goals investigated in this research were:

1. Generated output,
2. Cash dividends,

3. Fuel consumed, and
4. Cash borrowed.

Using a new computer program that was developed in this research, four test cases were studied. The results clearly showed the tradeoffs that a decision-maker must make and the cost for selecting one alternative over a different alternative.

Conclusions

In light of the investigation just completed, the following conclusions may be stated:

1. Goal programming is a desirable technique for a regulated industry facing conflicting objectives. The method of goal programming allows the decision-maker to explicitly examine the tradeoffs.
2. The goal programming methodology demonstrates how various rankings can change the timing of the cash flow needs of the utility.
3. The RGP program developed in this research provides preliminary data that indicates the CPU time to solve a goal programming model has been reduced. The program is designed to handle sparse matrices.

Recommendations

With regard to this research, some areas for future research are:

1. The ranking of the goals should be investigated from several viewpoints. A commission's ranking may not be compatible with the utility's ranking.
2. The model should be expanded to include the testing of replacement and depreciation policies.
3. The RGP program should be tested for increased efficiency. Currently, the program and the data reside in core. It may be more efficient to only read in data as required.

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ACKNOWLEDGMENTS

The author wishes to express his appreciation to Dr. Howard D. Meeks for his constant guidance and encouragement during my graduate studies and for his critical review of this dissertation.

I am indebted to Dr. Harold A. Cowles who helped the author define the research objectives and provided constructive criticism of this dissertation.

I am especially grateful to Dr. Keith L. McRoberts who gave me opportunities for professional growth and development.

I would also like to express my thanks to Dr. Vincent A. Sposito who advised the author during the early stages of the computer program development.

APPENDIX A: REVISED SIMPLEX METHOD WITH INVERSE
IN PRODUCT FORM

The problem being solved is

$$\text{minimize } z = cx$$

subject to

$$Ax = b, \quad x \geq 0$$

where

$$A = [P_1, P_2, \dots, P_m]$$

is an $m \times n$ matrix of rank m , b a $m \times 1$ vector of constants, and c a $1 \times n$ vector of objective coefficients. The equation

$$cx - z = 0$$

is added to the system, with $-z$ taken as an additional basic variable, here the $(m+1)$ st. Since the simplex is well-known, focus will be on the product form of the inverse. In what follows, it will be convenient to let the cost coefficient, c_i , be the $(m+1)$ st element of P_i and \bar{c}_i the $(m+1)$ st element of \bar{P}_i .

An elementary matrix is defined here as a square matrix differing from the identity in only one row or column. The inverse basis matrix, B^{-1} , is stored as a product of elementary matrices. Let B_c^{-1} be the current inverse and assume that the new inverse is to be computed by a pivot on

That is,

$$B_n = EB_c^{-1}$$

where B_n^{-1} is the new inverse. If the initial basis is the identity matrix, and if k pivot operations have been performed, the inverse at cycle k , B_k^{-1} , is given by

$$B_k^{-1} = E_k E_{k-1} \dots E_1$$

with each E_i an elementary matrix.

Recall that one of the steps in the simplex method is the selection of a nonbasic variable to enter the basis. The selection of a nonbasic variable is governed by

$$\bar{c}_j = c_j - c_B \bar{P}_j$$

where \bar{P}_j is the updated column and is determined as follows

$$\bar{P}_j = B^{-1} P_j$$

substitution yields

$$\bar{c}_j = c_j - c_B B^{-1} P_j$$

The product $c_B B^{-1}$ is called the simplex multipliers on the dual variables and is denoted by π . Using the product form, the simplex multipliers are given by

$$\begin{aligned}\pi &= c_B B^{-1} \\ &= (\dots((c_B E_K) E_{K-1}) \dots) E_1\end{aligned}$$

and the transformed column, \bar{P}_j , is given by

$$\begin{aligned}\bar{P}_j &= B^{-1} P_j \\ &= E_K(\dots(E_2(E_1 P_j)) \dots).\end{aligned}$$

An important property of elementary matrices is that they can be stored in a computer memory by recording only the elements of the nonunit vector column and its position in the matrix. These columns are often called "eta vectors". This is the procedure that was utilized in the computer program.

APPENDIX B: COMPUTER PROGRAM FOR REVISED GOAL
PROGRAMMING

CC
 CCC

THE PURPOSE OF THIS PROGRAM IS TO SOLVE LINEAR
 GOAL PROGRAMMING PROBLEMS WITH MULTIPLE OBJECTIVE

INPUT DATA

CARD 1 NDATA,MAXR,MAXC,MAXI FORMAT 4I5
 NUMBER OF NONZEROS IN THE A MATRIX, NUMBER OF
 ROWS AND COLUMNS IN THE A MATRIX, AND NUMBER
 OF ITERATIONS ALLOWED.

CARD 2 NTERMS, NOBJ, NVAR FORMAT 3I5
 NUMBER OF TERMS IN THE ACHIEVEMENT FUNCTION,
 NUMBER OF PRIORITY LEVELS, NUMBER OF DECISION
 VARIABLES.

CARD 3 ONE ELEMENT PER CARD
 JCOL,AMAT(J),IAMAT1(J) FORMAT I5,F10.5,5I5
 NUMBER OF COLUMN IN A MATRIX, VALUE OF ELEMENT,
 AND NUMBER OF ROW IN A MATRIX.

CARD 4 ONE CARD FOR EACH ELEMENT IN THE
 ACHIEVEMENT FUNCTION
 ICOL,ZMAT(J), IZMAT1(1,J), IZMAT1(2,J) FORMAT I5,F10.5,2I5
 ICOL IS THE PRIORITY LEVEL IN THE ACHIEVEMENT
 FUNCTION, ZMAT(J) IS THE VALUE IN THE ACHIEVEMENT
 FUNCTION, IZMAT1(1,J) IS THE PARTICULAR TERM IN
 THE PRIORITY LEVEL, AND IZMAT1(2,J) REPRESENTS
 +P OR -N VALUE

EXAMPLE N1 IS CODED AS -1, P2 CODED AS +2

CARD 5 ONE CARD FOR EACH RHS VALUE FORMAT 10.5
 RHS(I)

PRESENT ARRAY DIMENSIONS ACCOMMODATE 80 NONZERO
 ELEMENTS IN THE A MATRIX, 20 CONSTRAINTS, AND 10
 PRIORITY LEVELS AS A MAXIMUM, AND 40 TERMS IN
 THE ACHIEVEMENT FUNCTION.

CC
 CCC

DIMENSION AMAT(80),IAMAT1(80),IAMAT2(2,15),TMAT(120),
 * ITMAT1(120),

```

*          ITMAT2(3,25),ZMAT(40),IZMAT1(2,40),
*          IZMAT2(2,10),
*          RHS(20),IBASIC(20),ACHMT(10),OBJFCN(10,50),
*          NBASIC(30),TWOOF(10,20),TWNDF(10,30),
*          ANBSC(20,30),
*          TEMPT(20),AMT(20),TEMPA(20,30)
ITER=1
LAST=0
IAMAT2(1,1)=1
IAMAT2(1,2)=0
ITMAT2(1,1)=1
IZMAT2(1,1)=1
IZMAT2(1,2)=0
IPOB=1

C
C
C          READ DATA CARDS AND ZERO OUT
          THREE ROWS IN MATRIX

READ(5,5001) NDATA,MAXR,MAXC,MAXI
READ(5,5003) NTERMS,NOBJ,NVAR
DO 5 J=1,MAXC
  IAMAT2(2,J)=0
5  CONTINUE
DO 6 J=1,NOBJ
  IZMAT2(2,J)=0
6  CONTINUE
DO 7 J=1,MAXI
  ITMAT2(2,J)=0
7  CONTINUE
DO 10 J=1,NDATA
  READ(5,5002) JCOL,AMAT(J),IAMAT1(J)
  IAMAT2(2,JCOL)=1+IAMAT2(2,JCOL)
10 CONTINUE
DO 20 ICOL=2,MAXC
  IAMAT2(1,ICOL)=IAMAT2(1,(ICOL-1))+
  *IAMAT2(2,(ICOL-1))
20 CONTINUE
DO 30 J=1,NTERMS
  READ(5,5004) ICOL,ZMAT(J),IZMAT1(1,J),IZMAT1(2,J)
  IZMAT2(2,ICOL)=1+IZMAT2(2,ICOL)
30 CONTINUE
DO 40 K=2,NOBJ
  IZMAT2(1,K)=IZMAT2(1,(K-1))+IZMAT2(2,(K-1))
40 CONTINUE
READ(5,5005) (RHS(I),I=1,MAXR)

C
C
C          ALL DATA IS READ AND NOW ECHO
          CHECK FOR ANY ERRORS

WRITE(6,6001)

```

```

WRITE(6,6002) NDATA,MAXR,MAXC
WRITE(6,6003)
WRITE(6,6004) (AMAT(J),J=1,NDATA)
WRITE(6,6005) (IAMAT1(J),J=1,NDATA)
WRITE(6,6006) NOBJ,NTERMS
WRITE(6,6007)
WRITE(6,6008) (ZMAT(J),J=1,NTERMS)
DO 45 I=1,2
    WRITE(6,6009) (IZMAT1(I,J),J=1,NTERMS)
45 CONTINUE
WRITE(6,6010) (I,RHS(I),I=1,MAXR)

C
C     ESTABLISH THE COUNTER FOR INITIAL BASIC AND
C     NONBASIC VARIABLES. THE INITIAL BASIC
C     VARIABLES ARE THE NEGATIVE DEVIATES.
C
IVAR=MAXC
INEG=MAXC+MAXR
IPOS=MAXC+2*MAXR
DO 50 I=1,MAXR
    IBASIC(I)=IVAR+I
50 CONTINUE
DO 55 J=1,INEG
    IF(J.LE.IVAR) THEN DO
        NBASIC(J)=J
    ELSE DO
        NBASIC(J)=J+MAXR
    END IF
55 CONTINUE
MN=IPOS-MAXR

C
C
C     DETERMINE THE ANBSC MATRIX WHICH IS THE
C     A MATRIX FOR THE NONBASIC VARIABLE.
C
DO 90 I=1,MAXR
DO 80 IM=1,MN
    IF(NBASIC(IM).LE.IVAR) THEN DO
        LM=IAMAT2(1,IM)
        LN=IAMAT2(1,IM)+IAMAT2(2,IM)-1
        DO 70 ME=LM,LN
            IF(IAMAT1(ME).EQ.I) THEN DO
                ANBSC(I,IM)=AMAT(ME)
            GO TO 80
            END IF
70 CONTINUE
        ANBSC(I,IM)=0.0
        GO TO 80
    END IF

```



```

DO 240 JN=1,MN
  IF(NBASIC(JN).LE.IVAR) THEN DO
    TWNOF(1,JN)=0.0
    GO TO 240
  ELSE DO
    IF(NBASIC(JN).LE.INEG) THEN DO
      KL=-1*(NBASIC(JN)-IVAR)
    ELSE DO
      KL=NBASIC(JN)-INEG
    END IF
    DO 230 LM=1,NTERMS
      IF(IZMAT1(1,LM).EQ.I.AND.IZMAT1(2,LM).
*EQ.KL) THEN DO
        TWNOF(I,JN)=ZMAT(LM)
        GO TO 240
      ELSE DO
        IF(IZMAT1(1,LM).EQ.I.AND.IZMAT1(2,LM).
*EQ.KL) THEN DO
          TWNOF(I,JN)=ZMAT(LM)
          GO TO 240
        END IF
      END IF
      TWNOF(I,JN)=0.0
230 CONTINUE
    END IF
240 CONTINUE
260 CONTINUE

```

C
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C
C
C

THIS SECTION UPDATES THE OBJFCN
MATRIX AND THEN REPEATS THE PROCESS BY
RETURNING TO STATEMENT 105.

```

SUM=0.0
DO 330 I=1,NOBJ
DO 320 J=1,MN
DO 310 K=1,MAXR
  SUM=SUM+TWOF(I,K)*ANBSC(K,J)
310 CONTINUE
  OBJFCN(I,J)=SUM-TWNOF(I,J)
  SUM=0.0
320 CONTINUE
330 CONTINUE

```

C
C

```

DO 370 JK=1,NOBJ
  ACHMT(JK)=0.0
  DO 350 JM=1,MAXR

```

```

                                ACHMT(JK)=ACHMT(JK)+TWO*(JK,JM)*RHS(JM)
350                                CONTINUE
370 CONTINUE
C
C                                THIS STARTS THE MAIN LOOP OF THE
C                                PROGRAM. CHECK ON THE MAXIMUM NUMBER OF
C                                ITERATIONS ALLOWED. DETERMINES THE
C                                APPROPRIATE PRIORITY LEVEL TO MINIMIZE.
C
400 MA=IPOB
DO 410 I=MA,NOBJ
    IF(ACHMT(I).GT.0) THEN DO
        IPOB=I
        GO TO 415
    END IF
410 CONTINUE
C
C                                SELECT PIVOT COLUMN AND ENTERING VARIABLE
C                                IF ZCVAL STAYS AT ZERO THE PRIORITY
C                                LEVEL CAN NOT BE SATISFIED.
C
415 IF(IPOB.EQ.1) THEN DO
    ZCVAL=0.0
    DO 420 N=1,MN
        IF(OBJFCN(1,N).GT.ZCVAL) THEN DO
            ZCVAL=OBJFCN(1,N)
            IPCN=N
        END IF
420 CONTINUE
ELSE DO
    ZCVAL=0.0
    DO 425 N=1,MN
        IF(OBJFCN(IPOB,N).GT.ZCVAL) THEN DO
            MI=IPOB-1
            DO 423 NI=1,MI
                IF(OBJFCN(NI,N).LT.0.0) THEN DO
                    GO TO 425
                END IF
423 CONTINUE
                ZCVAL=OBJFCN(IPOB,N)
                IPCN=N
            END IF
425 CONTINUE
        IF(ZCVAL.EQ.0.0.AND.IPOB.EQ.NOBJ) GO TO 700
        IF(ZCVAL.EQ.0.0) THEN DO
            IPOB=IPOB+1
            GO TO 400

```

```

END IF
END IF
IF(ITER.GT.MAXI) GO TO 800

C
C
C
C
C
C
      SELECT PIVOT ROW AND LEAVING VARIABLE
      PITE VALUE IS DETERMINED

RATIO=16.5E12
DO 430 I=1,MAXR
  IF(ANBSC(I,IPCN).GT.0.0) THEN DO
    IF(RHS(I)/ANBSC(I,IPCN).LT.RATIO) THEN DO
      RATIO=RHS(I)/ANBSC(I,IPCN)
      IPRW=I
    END IF
  END IF
430 CONTINUE
IF(RATIO.LT.0.0) GO TO 850

C
C
C
C
C
      UPDATE THE LIST OF BASIC AND NONBASIC
      VARIABLES

ITEMP=IBASIC(IPRW)
IBASIC(IPRW)=NBASIC(IPCN)
NBASIC(IPCN)=ITEMP

C
C
C
C
C
      FOR THE PIVOT COLUMN DETERMINE THE
      ELEMENTARY TRANSFORMATION COLUMN.

PITE=ANBSC(IPRW,IPCN)
ITMAT2(3,ITER)=IPRW
DO 450 IK=1,MAXR
  IF(IK.EQ.IPRW) THEN DO
    TMAT(LAST+1)=1.0/PITE
    ITMAT2(2,ITER)=1+ITMAT2(2,ITER)
    ITMAT1(LAST+1)=IK
    LAST=LAST+1
  ELSE DO
    IF(ANBSC(IK,IPCN).NE.0.0) THEN DO
      TMAT(LAST+1)=-ANBSC(IK,IPCN)/PITE
      ITMAT2(2,ITER)=1+ITMAT2(2,ITER)
      ITMAT1(LAST+1)=IK
      LAST=LAST+1
    END IF
  END IF
450 CONTINUE

```

```

DO 460 ICOL=2,ITER
      ITMAT2(1,ICOL)=ITMAT2(1,(ICOL-1))+
      *ITMAT2(2,(ICOL-1))
460 CONTINUE

```

C
C
C
C
C

SWAP NEW NONBASIC COLUMN FOR OLD BASIC COLUMN

```

DO 490 I=1,MAXR
  IF(NBASIC(IPCN).LE.IVAR) THEN DO
    LM=IAMAT2(1,NBASIC(IPCN))
    LN=IAMAT2(1,NBASIC(IPCN))+
    *IAMAT2(2,NBASIC(IPCN))-1
    DO 470 ME=LM,LN
      IF(IAMAT1(ME).EQ.I) THEN DO
        ANBSC(I,IPCN)=AMAT(ME)
        GO TO 490
      END IF
    CONTINUE
    ANBSC(I,IPCN)=0.0
    GO TO 490
  ELSE DO
    IF(NBASIC(IPCN).LE.INEG) THEN DO
      IF((NBASIC(IPCN)-IVAR).EQ.I) THEN DO
        ANBSC(I,IPCN)=1.0
      ELSE DO
        ANBSC(I,IPCN)=0.0
      END IF
    ELSE DO
      IF((NBASIC(IPCN)-INEG).EQ.I) THEN DO
        ANBSC(I,IPCN)=-1.0
      ELSE DO
        ANBSC(I,IPCN)=0.0
      END IF
    END IF
  END IF
490 CONTINUE

```

C
C
C
C
C
C
C

THIS SECTION UPDATES THE ANBSC MATRIX, THE
ACHMT AND RHS VECTORS. THIS IS ACCOMPLISH
BY MULTIPLYING THE COLUMNS BY A SERIES OF
ELEMENTARY TRANSFORMATIONS.

```

DO 520 IJ=1,MAXR
  IC=ITMAT2(3,ITER)
  JI=ITMAT2(1,ITER)
  JK=ITMAT2(2,ITER)-1

```



```

JL=JI+JK
DO 515 JM=JI,JL
    IF(ITMAT1(JM).EQ.IJ) THEN DO
        TEMPT(IJ)=TMAT(JM)
        GO TO 520
    END IF
515 CONTINUE
    TEMPT(IJ)=0.0
520 CONTINUE
    DO 560 J=1,MAXR
        IF(J.EQ.IC) THEN DO
            DO 530 JK=1,MN
                TEMPA(J,JK)=TEMPT(J)*ANBSC(J,JK)
530 CONTINUE
            ELSE DO
                DO 540 JK=1,MN
                    TEMPA(J,JK)=ANBSC(J,JK)+TEMPT(J)*
*ANBSC(IC,JK)
540 CONTINUE
            END IF
560 CONTINUE
            DO 590 LI=1,MAXR
                IF(LI.EQ.IC) THEN DO
                    AMT(LI)=RHS(LI)*TEMPT(LI)
                ELSE DO
                    AMT(LI)=RHS(LI)+TEMPT(LI)*RHS(IC)
                END IF
590 CONTINUE
            DO 600 I=1,MAXR
                RHS(I)=AMT(I)
600 CONTINUE
            DO 640 I=1,MAXR
                DO 640 J=1,MN
                    ANBSC(I,J)=TEMPA(I,J)
640 CONTINUE
C
C
C
ITER=ITER+1
GO TO 100
C
C
C
WRITE FINAL RESULTS
700 WRITE(6,6301)
WRITE(6,6302)
DO 720 I=1,MAXR
    IF(IBASIC(I).LE.IVAR) THEN DO
        WRITE(6,6305) I,IBASIC(I),RHS(I)
    ELSE DO

```

```

        IF (IBASIC(I).LE.INEG) THEN DO
            WRITE(6,6303) I,IBASIC(I),RHS(I)
        ELSE DO
            WRITE(6,6304) I,IBASIC(I),RHS(I)
        END IF
    END IF
720 CONTINUE
    WRITE(6,6306)
    DO 760 K=1,NOBJ
        WRITE(6,6307) K,ACHMT(K)
760 CONTINUE
    ITER=ITER-1
    WRITE(6,6308) ITER
    GO TO 1000
800 WRITE(6,6401)
    GO TO 1000
850 WRITE(6,6402)

```

C
C
C
C
C
C

FORMAT STATEMENTS

```

5001 FORMAT(4I5)
5002 FORMAT(15,F10.3,I5)
5003 FORMAT(3I5)
5004 FORMAT(15,F10.3,2I5)
5005 FORMAT(F10.5)
6001 FORMAT('1',10X,12HPROBLEM DATA)
6002 FORMAT('0',10X,34HNUMBER OF NONZEROS IN THE A MATRIX,
    *I7,/'0',10X,30HNUMBER OF ROWS IN THE A MATRIX,I11,
    */'0',10X,33HNUMBER OF COLUMNS IN THE A MATRIX,I8)
6003 FORMAT('0',10X,63HTHE ORIGINAL DATA FOR THE MATRIX
    *OF COEFFICIENTS IS AS FOLLOWS,/'0',10X,28HTHE FIRST
    *ROW IS THE ELEMENT,/'0',10X,60HTHE SECOND ROW
    *IDENTIFIES THE PARTICULAR ROW FOR THE ELEMENT)
6004 FORMAT('0',(8F10.5,/' '))
6005 FORMAT('0',(8I10,/' '))
6006 FORMAT('0',10X,29HNUMBER OF OBJECTIVE FUNCTIONS,I12,
    */'0',10X,41HNUMBER OF TERMS IN THE OBJECTIVE FUNCTION
    *,I7)
6007 FORMAT('0',10X,70HTHE ORIGINAL DATA FOR THE MATRIX
    *OF OBJECTIVE FUNCTIONS IS AS FOLLOWS,/'0',10X,54H
    *THE FIRST ROW IS THE WEIGHT IN THE OBJECTIVE FUNCTION
    *S,/'0',10X,76HTHE SECOND ROW IDENTIFIES THE
    *PARTICULAR OBJECTIVE FUNCTION FOR THAT WEIGHT)
6008 FORMAT('0',(8F10.5,/' '))
6009 FORMAT('0',(8I10,/' '))
6010 FORMAT('0',10X,48HTHE ORIGINAL RESOURCE REQUIREMENT IS
    *AS FOLLOWS,/'0',(10X,3HROW,I4,F10.0))

```

```
6301 FORMAT('1',34HTHE OPTIMAL SOLUTION IS AS FOLLOWS)
6302 FORMAT('0',10X,15HBASIC VARIABLES,24X,4HTYPE,24X,
*5HVALUE)
6302 FORMAT('0',24X,15HBASIC VARIABLES,24X,4HTYPE,24X,5HVAL
6303 FORMAT('0',131,31X,3HNEG,13,21X,F12.2)
6304 FORMAT('0',131,31X,3HPOS,13,21X,F12.2)
6305 FORMAT('0',131,31X,1HX,15,21X,F12.2)
6306 FORMAT('0',33X,14HPRIORITY LEVEL,33X,5HVALUE)
6307 FORMAT('0',141,33X,F12.2)
6308 FORMAT('0',37HTHE NUMBER OF ITERATIONS REQUIRED
*WAS .15)
6401 FORMAT('1',10X,38HMAXIMUM NUMBER OF ITERATIONS
*EXCEEDED.)
6402 FORMAT('1',36HRATIO TEST FAILED, PROBLEM UNBOUNDED)
1000 STOP
END
```